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UNSCENTED KALMAN FILTER APPLICATION FOR STATE ESTIMATION OF A QBALL-X4 QUADROTOR

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ABSTRACT

In the process of controlling the quadrotor, accurate state estimation plays a crucial role. For positioning purposes, the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) are employed to determine the position of moving subjects. The Qball-X4 quadrotor is a highly nonlinear object, and when combined with Gauss interference, it can compromise the accuracy of the EKF. To address these challenges, this study focuses on assessing the suitability of the UKF nonlinear filtering method for estimating the state of the Qball-X4 quadrotor. This estimation is based on measurements from the gyroscope and Global Positioning System (GPS). To simulate real-life conditions, measurement noise has been deliberately introduced into the sensors. Rigorous testing under various conditions has emphasized the superior performance of the UKF filter in estimating the state of the quadrotor. This paper presents a valuable method to enhance the accuracy and reliability of the navigation system for the Qball-X4 quadrotor.

Keywords: kalman filter, measurement noise, position estimation, Qball-X4 quadrotor.

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INTRODUCTION

Introducing the Qball-X4 Type Quadrotor

Unmanned aerial vehicles (UAVs) have seen a surge in applications owing to their structural simplicity, affordability, and ease of operation. This paper delves into the study of a Qball-X4 quadrotor, powered by four propeller motors, with Data Acquisition (DAQ) cards gathering data from the HiQ compartment. The GPS receiver is seamlessly integrated with the HiQ daughterboard through a GPS serial input labeled ID10. Moreover, the quadrotor is equipped with real-time control software (QuaRC), enabling researchers to swiftly develop and test controllers using the Matlab/Simulink interface. Controller models crafted in Simulink are easily transferred and compiled into executable files on the Gumstix-embedded computer using QuaRC 1. The system configuration is visually represented in Figure-1.

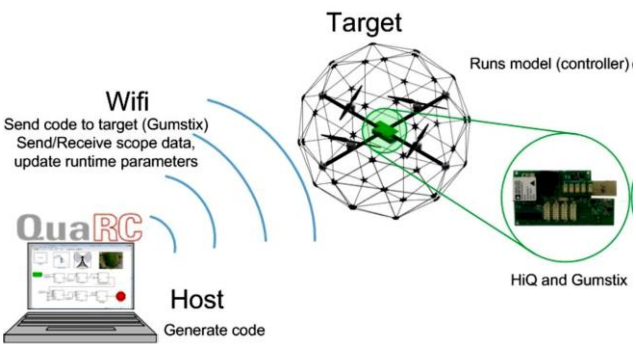

Figure-1. Configuration of the Qball-X4 type quadrotor.

Consider the relevant reference frames illustrated in Figure-2, where Oxyz represents the inertial reference frame, and BXYZ is the relative reference frame attached to the Quadrotor, corresponding to the fixed frame of the Quadrotor. The three rotation angles of the Quadrotor around the corresponding axes are denoted as the roll angle Φ, the pitch angle θ, and the yaw angle Ψ. Furthermore, F_1, F_2, F_3, and F_4 represent the thrust generated by the four propellers.

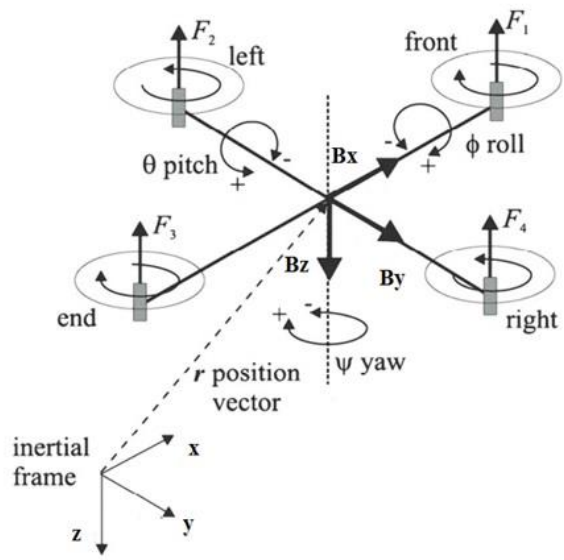

Figure-2. Quadrotor reference frames.

Introduction to Kalman Filters

The estimation algorithm for the system of nonlinear equations used in positioning has been studied for many years. In 1960, Kalman proposed an algorithm known as Kalman filtering, which was applied to linear equations with noise obeying Gauss's law of normal distribution. However, most of the equations used are practically non-

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linear, so this calculation technique is not yet applicable. To overcome these shortcomings, Anderson and Moore developed a new method in 1979 based on an extension of Kalman's filtering algorithm called EKF. The EKF filter requires linearization for nonlinear equations. However, linearization becomes less accurate when applied to highly nonlinear equations. To address this limitation, in 1997, Julier and Uhlmann proposed the UKF filter, which eliminates the need for linearization steps. The accuracy of this method is comparable to the EKF when utilizing second-order coefficients in the Taylor expansion.

LITERATURE REVIEW

The following papers introduce the UKF non-linear Kalman filtration method and some of its applications:

The paper [2] introduced the UKF method for estimating the state of non-linear systems. The paper points out the limitations of the EKF method and proposes a new method based on the use of sample points to approximate the probability distribution of the state. The paper also compares the effectiveness of UKF and EKF.

The paper [3] gave an overview of the UKF method and its applications in various fields, such as control, positioning, and identification. The paper also covers UKF variants, such as square-root UKF, divided difference UKF, and central difference UKF.

The paper [4] delves into the study of the application of UKF to the estimation of both states and parameters of non-linear systems. The paper proposes a method called dual estimation, which combines UKF and expectation maximization algorithms. The paper also illustrates the results of this method for estimating the state of a pendulum.

The paper [5] delves into the application of the UKF for estimating the dynamic states of power systems. It argues that the application of UKF holds the potential to effectively address challenges posed by non-linearity, latency, and interference within power systems.

The paper [6] explored a variant of the UKF known as the square-root UKF. This method employs Cholesky analysis to calculate sample points, offering advantages such as minimizing rounding errors and enhancing the overall stability of the UKF.

The paper [7] delves into a novel method, the unscented transformation, designed for transforming non-linear probability distributions. This method serves as the foundational concept for the UKF filter and finds applications in various filtering and estimation problems.

The paper [8] explored the application of UKF to address probabilistic inference problems in dynamic-state space models. It introduces a method known as the sigma-point Kalman filter, which combines elements of UKF and particle filters. The paper provides illustrative results from applying this method to estimate the state of a mobile robot. Additionally, it showcases the significance of the method in training a hidden Markov model (HMM).

The paper [9] delves into the application of the sigma-point Kalman filter to state estimation problems and sensor fusion to non-linear systems such as cars.

The paper [10] delves into the application of UKF to simultaneous localization and mapping (SLAM) on a drone. The paper uses a dynamical model based on quaternion to represent the state of the drone and uses an image sensor to collect features of the environment.

As such, papers [2] to [11] share the commonality of focusing on the UKF method and its applications to non-linear systems. However, these papers do not yet include content on the application of UKF to the estimation of the state of a Qball-X4 quadrotor.

In this paper, the author proposes the utilization of the UKF method to accurately and stably estimate the state of the Qball-X4, a type of quadrotor known for its unique features and compatibility with various sensor types. The novelty and creativity of the study lie in the author's demonstration that the application of the UKF method can yield accurate and stable state estimates, even in the presence of measurement noise affecting GPS sensors and gyroscopes.

MODEL OF THE QBALL-X4 QUADROTOR

Equations of Motion

The Qball-X4 Quadrotor is a simple-structured drone with the features of four motors mounted in a cross-shaped structure, each equipped with a propeller. The 'front-rear' propeller rotates counterclockwise while the 'right-left' propeller rotates clockwise. This configuration is essential for ensuring the stable operation of the quadrotor [1], as it helps in achieving balanced lift and control.

$$F_i = K_f \omega_i^2$$

The thrust (F_i) generated by each propeller is determined by the thrust coefficient (K_f) and the angular velocity of the propeller (ω_i), and in addition to upward thrust, each rotating propeller produces torque calculated by the formula:

$$M_i = K_m \omega_i^2$$

where M_i is the moment generated by each blade along the z-axis and K_m is the drag coefficient.

To find the dynamic equations of the system, start with the expression for calculating the total thrust of the four blades:

$$F_\Sigma = K_f(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)$$

The linear acceleration equations are described as follows:

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$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix}=\begin{bmatrix}0\\0\\-g\end{bmatrix}+\frac{R}{m}\begin{bmatrix}0\\0\\F_\Sigma\end{bmatrix};\ R=R_z(\psi)R_y(\theta)R_x(\phi)$$

Where *R* is the rotation matrix from the coordinate system attached to the earth to the corresponding coordinate system according to the axes of the quadrotor. The force moments along the corresponding axes are as follows:

$$\begin{cases}\tau_x=\frac{LK_f}{\sqrt{2}}\left(\omega_1^2+\omega_2^2-\omega_3^2-\omega_4^2\right)\\ \tau_y=\frac{LK_f}{\sqrt{2}}\left(-\omega_1^2+\omega_2^2+\omega_3^2-\omega_4^2\right)\\ \tau_z=K_m\left(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2\right)\end{cases}$$

The angular acceleration associated with the quadrotor is described by the Euler equation within the dynamic model. This equation is formulated as follows:

$$\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix}=I^{-1}\left(\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix}-\omega\times I\omega\right);\ I=\begin{bmatrix}I_{xx}&0&0\\0&I_{yy}&0\\0&0&I_{zz}\end{bmatrix};\ \omega=\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$$

From the above relationships, the equations of motion are written as follows:

$$\begin{cases}\ddot{x}=\frac{F_\Sigma\left(\cos(\phi)\sin(\theta)\cos(\psi)+\sin(\phi)\sin(\psi)\right)}{M}\\ \ddot{y}=\frac{F_\Sigma\left(\cos(\phi)\sin(\theta)\sin(\psi)-\sin(\phi)\cos(\psi)\right)}{M}\\ \ddot{z}=\frac{F_\Sigma\left(\cos(\phi)\cos(\theta)\right)}{M}-9.81\\ \ddot{\phi}=\frac{\tau_x+\left(I_{yy}-I_{zz}\right)\dot{\theta}\dot{\psi}}{I_{xx}}\\ \ddot{\theta}=\frac{\tau_y+\left(I_{zz}-I_{xx}\right)\dot{\phi}\dot{\psi}}{I_{yy}}\\ \ddot{\psi}=\frac{\tau_z+\left(I_{xx}-I_{yy}\right)\dot{\theta}\dot{\phi}}{I_{zz}}\end{cases}$$

Based on the input angular velocity of the rotor, it is possible to derive the continuous state of the quadrotor, including position and direction, according to the input angular velocity of the rotor, thrust force, and force moment around the corresponding axes. The position and direction, measurable using a GPS device, are continuous states of the quadrotor. On the other hand, the force moment around the corresponding axes cannot be physically measured and requires estimation. GPS measurements, while providing valuable data, are subject to noise, rendering them non-absolute.

The continuous-time state-space model of the Qball-X4 is as follows:

$$\dot{X}=f_x(X,u)+n$$

Where *u* is the input vector, *X* is the state vector of the system, $f_x(X,u)$ is the nonlinear function matrix, and *n* is the process noise or input noise.

$$\begin{cases}X=\begin{bmatrix}x & y & z & \phi & \theta & \psi & \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi}\end{bmatrix}\\ u=\begin{bmatrix}F_\Sigma & \tau_x & \tau_y & \tau_z\end{bmatrix}\end{cases}$$

The process noise or input noise is $n \sim N(\bar{x},\sigma_\omega)$, assumed to be Gaussian distributed with a mean of $\bar{x}=0$ and a variance of $\sigma_\omega=1e-6$.

Model of the Sensor

GPS measurement sensor to measure the position of the quadrotor in a spherical coordinate system. GPS sensors have the following mathematical model:

$$y_k^{GPS}=r(kT_s)+n_k^{GPS}$$

In there,

y_k^{GPS} is the GPS measurement.

$r(kT_s)$ is the actual position of the quadrotor in spherical coordinates.

n_k^{GPS} is the measurement noise with a Gaussian distribution of $n_k^{GPS} \sim N(0,\sigma_{GPS})$ with a standard deviation of σ_{GPS}.

A gyroscope to measure angular velocity in a coordinate system attached to the quadrotor. The gyroscope has the following mathematical model:

$$y_k^{Gyro}=\dot{\Omega}(kT_s)+n_k^{Gyro}$$

In there,

y_k^{Gyro} is the gyroscope measurement.

$\dot{\Omega}(kT_s)$ is the actual angular velocity of the quadrotor.

n_k^{Gyro} is the measurement noise with a Gaussian distribution of $n_k^{Gyro} \sim N(0,\sigma_{Gyro})$ with a standard deviation of σ_{Gyro}.

APPLYING THE UKF ALGORITHM TO QBALL-X4

The UKF Filter excels in non-linear model cases as it doesn't require assumptions about the linearity of the system or measurement model. It is robust even when derivative conditions are unstable or measurement errors are large. Hence, the UKF filter is frequently used for estimating the state of systems with uncertain models or non-linear measurements.

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The primary objective of this study is to employ the UKF filter to precisely estimate the position of the Qball-X4 quadrotor. This involves mitigating the noise inherent in GPS measurements and deducing orientation values that cannot be directly measured. The application of the UKF filter is pivotal in achieving accurate and stable estimates in the presence of measurement uncertainties and non-linearities.

The Unscented Kalman Filter employs a deterministic sampling method to effectively represent the probability distribution of a state by utilizing a set of carefully selected sigma points. These sigma points are chosen to accurately preserve the mean and covariance of the probability distribution. Subsequently, they undergo transformation through the nonlinear function, producing new points that represent the probability distribution of the next state. Ultimately, the mean and covariance of the updated probability distribution are computed with the application of weighted values to the sample points.

In the paper, the UKF was selected to filter sensor measurements, playing a key role in estimating the state of the quadrotor. Given the noise in GPS measurements, it is essential to filter out this noise. Directly measuring the direction of the quadcopter is impossible, making it an unobservable state in the system. The UKF estimates this unobservable state based on gyroscope measurements, effectively filtering out noise.

Time update:

$$x^{(i)} = \hat{x}^{(+)} + \tilde{x}^{(i)}$$

$$x_{k+1}^{(i)} = f\left(x^{(i)}, u_k\right)$$

$$\hat{x}_{k+1}^{(-)} = \frac{1}{2n}\sum_{i=1}^{2n} x_{k+1}^{(i)}$$

$$P_{k+1}^{(-)} = \frac{1}{2n}\sum_{i=1}^{2n}\left(x_{k+1}^{(i)} - \hat{x}_{k+1}^{(-)}\right)\left(x_{k+1}^{(i)} - \hat{x}_{k+1}^{(-)}\right)^T + Q$$

Measurement update:

$$\bar{Y} = \frac{1}{2n}\sum_{i=1}^{2n} Y^{(i)}$$

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n}\left(x_k^{(i)} - \hat{x}_k^{(-)}\right)\left(Y^{(i)} - \bar{Y}\right)^T$$

$$P_y = \frac{1}{2n}\sum_{i=1}^{2n}\left(Y^{(i)} - \bar{Y}\right)\left(Y^{(i)} - \bar{Y}\right)^T + R$$

$$K = P_{xy}P_y^{-1}$$

$$x_k^{(+)} = \hat{x}_k^{(-)} + K\left(Y_k - \bar{Y}\right)$$

$$P_k^{(+)} = P_k^{(-)} - KP_{xy}^T$$

Thus, UKF's main implementation steps in the Matlab program can be summarized as follows:

- Step 1: Select a set of sample points from the probability distribution of the current state.

```
M = chol(P_k1,'upper');
xBar_i_p = sqrt(n)*[M -M];
xI_p = x_k0 + xBar_i_p;
```

In there,

M is the Cholesky matrix of the covariance matrix P_k1.

xBar_i_p is the set of sample points generated from the Cholesky matrix.

xI_p is the set of new sample points.

- Step 2: Pass each sample point through the nonlinear function of the dynamic system, obtaining a set of new sample points belonging to the probability distribution of the next state.

```
for i = 1:(2*n)
x_ki(:,i) = simout_Temp.simout.Data(end,:)';
end
```

In it, each sample point from xI_p is passed through the system model to obtain x_ki, a set of new sample points belonging to the probability distribution of the next state.

- Step 3: Calculate the weights for the sample points.

```
xCap_Minus = (1/(2*n))*sum(x_ki,2);
```

where xCap_Minus is the average value of the set of sample points.

- Step 4: Calculate the mean and covariance of the probability distribution of the next state using the weights and new sample points.

```
PMinus = zeros(12);
for i = 1:(2*n)
PMinus=PMinus+(x_ki(:,i)-xCap_Minus)*(x_ki(:,i)-
xCap_Minus)' +Q;
end
PMinus = (1/(2*n)).*PMinus;
```

where PMinus is the covariance matrix of the next state.

- Step 5: Update the mean and covariance of the probability distribution of the next state using measured observations and Kalman-Gain formulas.

```
M = chol(PMinus,'upper');
xBar_i = sqrt(n)*[M -M];
xI_p = xCap_Minus + xBar_i;
for i = 1:(2*n)
yMeasure(:,i)= simout_Temp.sensorMeasure.Data(end,:)';
end
yBar = (1/(2*n))*sum(yMeasure,2);
Pxy = zeros(12,6);
Py = zeros(6);
for i = 1:(2*n)
Pxy=Pxy+(xI_p(:,i)-xCap_Minus)*(yMeasure(:,i)-yBar)';
Py = Py + (yMeasure(:,i)-yBar)*(yMeasure(:,i)-yBar)' + R;
end
Pxy = (1/(2*n)).*Pxy;
Py = (1/(2*n)).*Py;
K = Pxy*inv(Py);
xCap = xCap_Minus + K*(yMeasurement(:,count) - yBar);
P_k = PMinus - K*Pxy';
```

In there,

xBar_i is the set of new sample points belonging to the probability distribution of the next state.

yBar is the average value of the set of measurements.

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Pxy and Py are covariance matrices related to the correlation between state and measurement.

K is the Kalman-Gain matrix.

xCap and P_k are the mean and covariance of the probability distribution of the next state after obtaining measurement data.

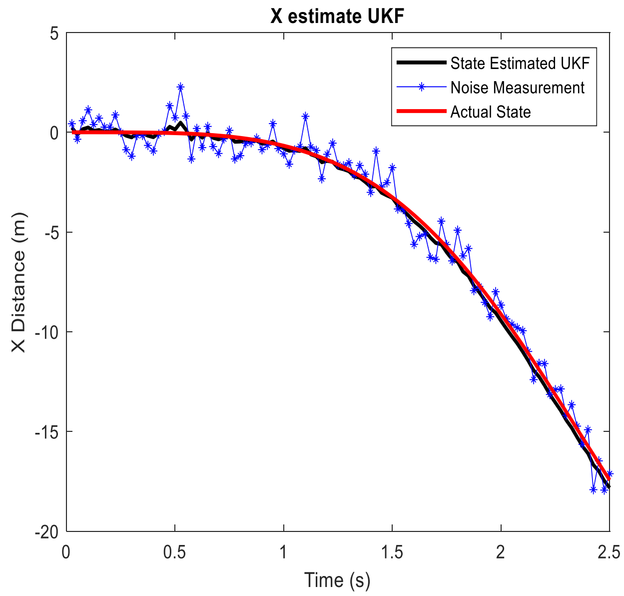
RESULTS AND DISCUSSIONS

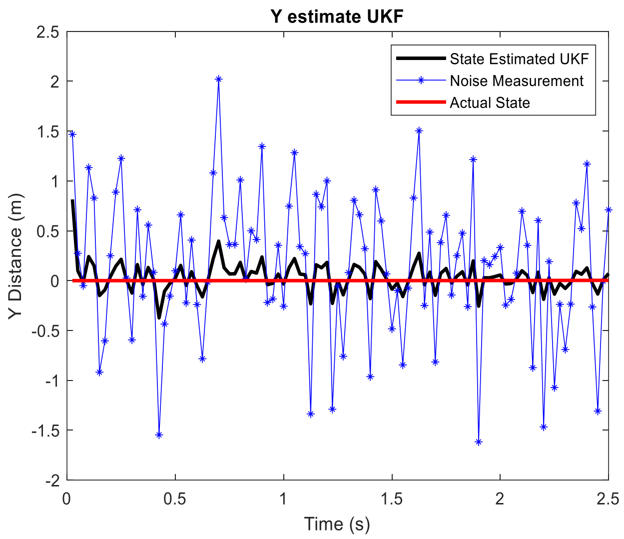
The Qball-X4 quadrotor was modeled in Simulink with the parameters shown in Table-1 and simulated for 2.5 seconds with time step $T_s = 0.1(s)$.

The plots obtained in Figure-3 and Figure-4 are estimates of the position coordinates (x, y, and z) and angular coordinates (roll angle Φ, pitch angle θ, and yaw angle Ψ) obtained from UKF implementation.

Table-1. System variables.

Parameter	Symbol	Value	Unit
Mass	M	1,4	kg
Moments of inertia about the x, y, and z axes	I_{xx}, I_{yy}, I_{zz}	0,03	$Kg.m^2$
The distance between the propeller and the center of gravity	L	0,2	m
Thrust factor	K_f	0,1	
Drag factor	K_m	0,1	
Angular velocity of the propeller the anuglar velocity of each propeller	ω_i i = 1÷4	2,8; 3,2; 3,2; 2,8	rad/s
Standard deviation of GPS measurement noise	σ_{GPS}	0,54	m
The standard deviation of the measurement noise from the gyroscope	σ_{Gyro}	0,15	radian





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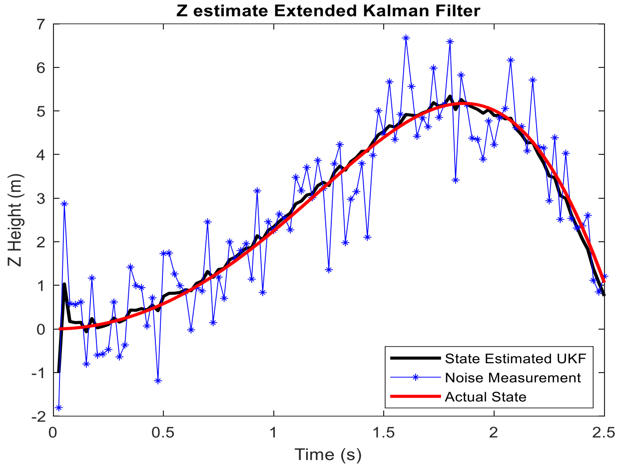
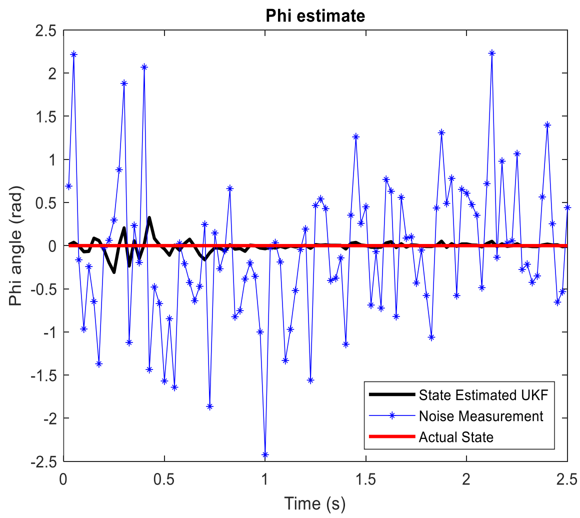
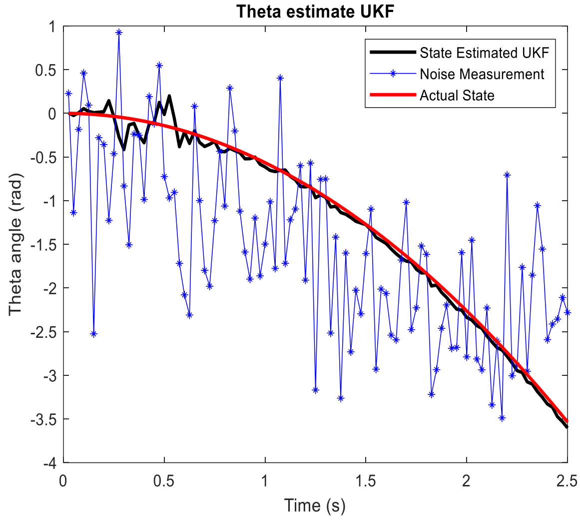


Figure-3. The position coordinates obtained from UKF implementation.

Position (x, y, z) and angular (roll Φ, pitch θ, yaw Ψ) coordinates are estimated using the UKF filter, and the results are evaluated with Figure-3 and Figure-4. The red line in each figure represents the actual state of the quadrotor's position and angular coordinates. Without a UKF filter, the measured quadrotor's state fluctuates sharply along the blue line due to measurement noise on the GPS sensor and gyroscope. The UKF filter largely excludes the effect of measurement noise on the GPS sensor and gyroscope. Consequently, the state of the measured quadrotor varies along the black line, converging with the red line.

Through angular coordinate state graphs, the UKF proves highly effective in predicting quadrotor angular states. This highlights strength of the UKF, which is particularly beneficial as non-linearity is more prominent in measuring angular coordinates. It underscores the suitability of the UKF for complex, nonlinear systems. The UKF filter ensures stable, high-quality operation of the quadrotor, even in the presence of measurement noise on the GPS sensor and gyroscope.





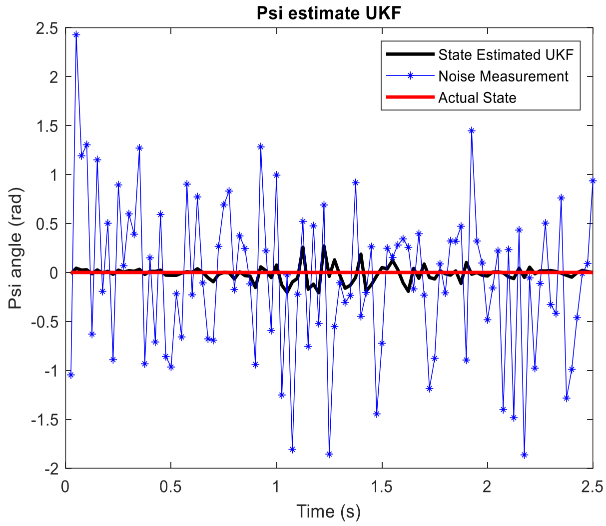


Figure-4. The angular coordinates obtained from UKF implementation.

CONCLUSIONS

This study focuses on applying the UKF filter to estimate the state of the Qball-X4 quadrotor during control. The UKF effectively addresses challenges posed by the highly nonlinear nature of the Qball-X4 and the impact of measurement noise on GPS sensors and gyroscopes. During the estimation of position and angular coordinates, the UKF not only reduces quadrotor state oscillation caused by measurement noise but also nearly eliminates it. As a result, there is stable convergence between the measured and actual states, particularly under non-linear conditions and the influence of measurement noise from GPS sensors and gyroscopes.

In addition, the analysis of the angular coordinate state graph showcases the prowess of UKF in predicting quadrotor angular states, particularly in highly nonlinear environments. This underscores the effectiveness of UKF as a powerful tool for state estimation in nonlinear and complex systems such as quadrotors.

In conclusion, this study not only offers an effective state estimation method for the Qball-X4

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quadrotor but also paves the way for significant applications of the UKF in non-linear control systems. This has the potential to enhance the reliability and accuracy of navigation systems in practical quadrotor applications.

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