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IMPACT OF HALL ON PERISTALTIC FLOW OF A NEWTONIAN FLUID IN A VERTICAL CHANNELTHROUGH A POROUS MEDIUM

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ABSTRACT

A study is carried out to examine the influence of Hall effects on the peristaltic flow of Newtonian fluid through porous medium under the assumption of long wave length in a two- dimensional vertical channel. The non-dimensional flow governing equations with boundary conditions are solved for axil velocity and axial pressure gradient. The influence of various emerging parameters on the pumping characteristics are studied and explained with the aid of graphs.

Keywords: darcy number, hall, MHD, Newtonian fluid, peristaltic flow.

INTRODUCTION

A branch of science which deals with properties magnetism and electrically conducting fluids such as electrolytes, plasmas and salt water is called magneto hydrodynamics (MHD). In magneto hydro dynamics, magnetic fields can allow currents in a fluid which is electrically conducted, and it separates the fluid and changes the magnetic field. MHD is designed with the amalgam of Maxwell's equations of electromagnetism and Navier-Stokes differential equations of fluid dynamics. Solution of these equations can be found either numerically or analytically. The impact of moving magnetic field on blood stream was concentrated by Agrawal and Anwaruddin (1984) and examined that an increase in magnetic field leads to an enrichment in fluid velocity. Elshahed and Haroun (2005) deliberate the effect of magnetic field on peristaltic flow of Johnson-Segalman fluid. Subba Narasimhudu and Subba Reddy (2017) have done their research to study the effect of Hall on the peristaltic flow of a Newtonian fluid in a channel.

Moreover, the number of researchers have been investigated the fluid flow through a porous medium by using Darcy's law Scheidegger (1974) and Varshney (1979) and Raptis and Perdikis (1983) have given some studies on this point. The first research work was presented by Elsehawey *et al.* (1999) on peristaltic flow through a porous medium and also examined the peristaltic motion of a generalized Newtonian fluid through a porous medium (2000).

By taking above works into account, we examine the effect of Hall on the peristaltic stream of a Newtonian fluid through a permeable medium in a two-dimensional vertical channel under the presumption of long wavelength. A closed form solution is acquired for axial velocity and pressure gradient. The impacts of many arising parameters on the time- averaged volume flow rate are examined with the help of graphs.

FORMULATION OF THE PROBLEM

We assumed the peristaltic pumping of a conducting Newtonian fluid flow through a porous medium in a vertical channel of half-width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For convenience, we confined our discussion to the half-width of the channel as shown in the Figure-1. The wall deformation is given by

$$H(X,t) = a + b\sin\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{2.1}$$

Here b represents the amplitude, λ the wavelength and c is the wave speed.

Under the presumptions that the channel length is an integral multiple of the wavelength $\lambda$and the pressure contrast across the ends of the channel is a consistent, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X,Y). The change between these two casings is given by

$$x = X - c\,t,\ y = Y,\ u = U - c,\ v = V \text{ and } p(x) = P(X,\,t) \tag{2.2}$$

Where $(u,\ v)$ and $(U,\ V)$ are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

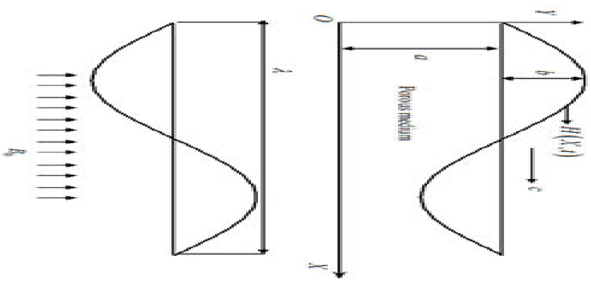

Figure-1. Physical pattern.

The flow governing equations in wave frame are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.3}$$

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$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B_0^2}{1+m^2}\left(mv-\left(u+c\right)\right)-\frac{\mu}{k}\left(u+c\right)+\rho g \tag{2.4}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B_0^2}{1+m^2}\left(m\left(u+c\right)+v\right)-\frac{\mu}{k}v-\rho g \tag{2.5}$$

where ρ is density, g is acceleration due to gravity, σ is electrical conductivity, B_0 is the magnetic field strength, m is Hall parameter, k is permeability of the porous medium.

The boundary conditions dimensional form are

$$u=-c \quad \text{at} \quad y=H \tag{2.6}$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \tag{2.7}$$

Presenting the dimensionaless quantities

$$x^{'}=\frac{x}{\lambda},\ y^{'}=\frac{y}{a},u^{'}=\frac{u}{c},v^{'}=\frac{v}{c\delta},\delta=\frac{a}{\lambda},p^{'}=\frac{pa^2}{\mu c\lambda},t^{'}=\frac{ct}{\lambda},h=\frac{H}{a},\ \phi=\frac{b}{a},q^{'}=\frac{q}{ac},M^2=\frac{\sigma a^2 B_0^2}{\mu},Da=\frac{k}{a^2}$$

Into equations (2.3) to (2.5), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.8}$$

$$\mathrm{Re}\,\lambda\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\lambda^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{M^2}{1+m^2}\left(m\lambda v-\left(u+1\right)\right)-\frac{1}{Da}\left(u+1\right)+\frac{\mathrm{Re}}{Fr} \tag{2.9}$$

$$\mathrm{Re}\,\lambda^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\lambda^2\left(\lambda^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\lambda M^2}{1+m^2}\left(m\left(u+1\right)+\lambda v\right)-\frac{\lambda^2}{Da}v-\frac{\mathrm{Re}}{Fr}\lambda \tag{2.10}$$

Here Re is the Reynolds number, M is the Hartmann number, $Fr=\dfrac{c^2}{ag}$ is the Froude number, and Da is the Darcy number.

Using extended wavelength (i.e., $\lambda << 1$) approx., the equations (2.9) and (2.10) become

$$\frac{\partial^2 u}{\partial y^2}-N^2 u=\frac{\partial p}{\partial x}-\frac{\mathrm{Re}}{Fr}+N^2 \tag{2.11}$$

$$\frac{\partial p}{\partial y}=0 \tag{2.12}$$

where $N=\sqrt{\dfrac{M^2}{1+m^2}+\dfrac{1}{Da}}$.

From equation (2.12), it is understood that p is function of x. Then (2.11) can be written as

$$\frac{\partial^2 u}{\partial y^2}-N^2 u=\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}+N^2 \tag{2.13}$$

The boundary conditions in non-dimensional form are given as

$$u=-1 \quad \text{at} \quad y=h=1+\phi\sin 2\pi x \tag{2.14}$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \tag{2.15}$$

Knowing the velocity, the volume flow rate q in a wave frame is given by

$$q=\int_0^h u\,dy \tag{2.16}$$

The instantaneous flow $Q(X,t)$ in the laboratory frame is

$$Q(X,t)=\int_0^h U\,dY=\int_0^h\left(u+1\right)dy=q+h \tag{2.17}$$

The $\overline{Q}$ over one period $T\left(=\dfrac{\lambda}{c}\right)$ of the peristaltic wave is given by

$$\overline{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{2.18}$$

SOLUTION

Cracking equation (2.13) together with boundary conditions (2.14) and (2.15), we get

$$u=\frac{1}{N^2}\left(\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\right)\left[\frac{\cosh Ny}{\cosh Nh}-1\right]-1 \tag{3.1}$$

The volume flow rate q in a wave frame of reference is given by

$$q=\frac{1}{N^3}\left(\frac{dp}{dx}-\frac{\mathrm{Re}}{Fr}\right)\left[\frac{\sinh Nh-Nh\cosh Nh}{\cosh Nh}\right]-h \tag{3.2}$$

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From equation (3.3), we write

$$\frac{dp}{dx} = \frac{(q+h)N^3 \cosh Nh}{\sinh Nh - Nh \cosh Nh} + \frac{\text{Re}}{Fr} \tag{3.3}$$

The dimensionless pressure rise Δp per one wavelength is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \tag{3.4}$$

As $\text{Re} \to 0$ and $Da \to \infty$, our outcomes coincides with the consequences of Subbanarasimhudu and Subba Reddy (2017).

RESULT AND DISCUSSIONS

Figure-2 depicts the difference of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for various values of Hartmann number M with $Da = 0.1$, $\text{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $m = 0.2$. It is identified that, the time-averaged flow rate $\overline{Q}$ increases in the pumping region with rising values of M, while it decreases in both the free-pumping and co-pumping regions with increasing M.

The Δp with $\overline{Q}$ for diverse values of Hall parameter m with $Da = 0.1$, $\text{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $M = 1$ is portrayed in Figure-3. It is noticed that, the $\overline{Q}$ declines in the pumping region and upsurges in both the free-pumping and co-pumping regions on cumulative m.

Figure-4 demonstrates the Δp with $\overline{Q}$ for different values of Da with $m = 0.2$, $\text{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $M = 1$. It is understood that, the $\overline{Q}$ decreases in the pumping region and increases in both the free-pumping and co-pumping regions with increasing Da.

The Δp with $\overline{Q}$ for dissimilar values of Reynolds number Re with $m = 0.2$, $Da = 0.1$, $Fr = 2$, $\phi = 0.5$ and $M = 1$ is Figure-5. It is spotted that, $\overline{Q}$ increases with increasing Re in all the three regions.

Figure-6 depicts the Δp with $\overline{Q}$ for altered values of Fr with $m = 0.2$, $\text{Re} = 0.5$, $Da = 0.1$, $\phi = 0.5$ and $M = 1$. It is detected that, the $\overline{Q}$ decreases with increasing Fr in all the three regions.

The Δp with $\overline{Q}$ for distinctive values of ϕ with $Da = 0.1$, $\text{Re} = 0.5$, $Fr = 2, M = 1$ and $m = 0.2$ is exposed in Fig. 7. It is found that that the $\overline{Q}$ increases with increasing ϕ in pumping and free pumping regions and decreases in the co-pumping region.

CONCLUSIONS

In this article, we explored the effect of Hall on the peristaltic flow of a fluid through a porous medium in a vertical two-dimensional channel under the assumption of long wavelength approximation. An analytical method adopted to find the solution for the velocity field and pressure gradient. It is perceived that, the $\overline{Q}$ in the pumping region is increases with increasing values of M, Re and ϕ, while it experiences declinement with increasing m, Da and Fr.

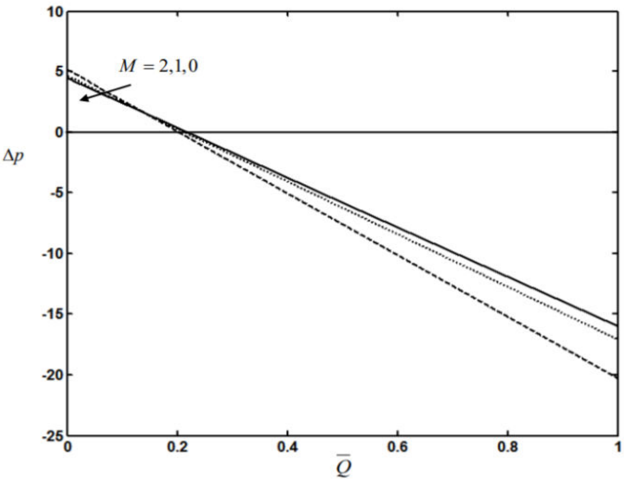

Figure-2. The Δp with $\overline{Q}$ for various estimations of M with $Da = 0.1$, $\text{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $m = 0.2$.

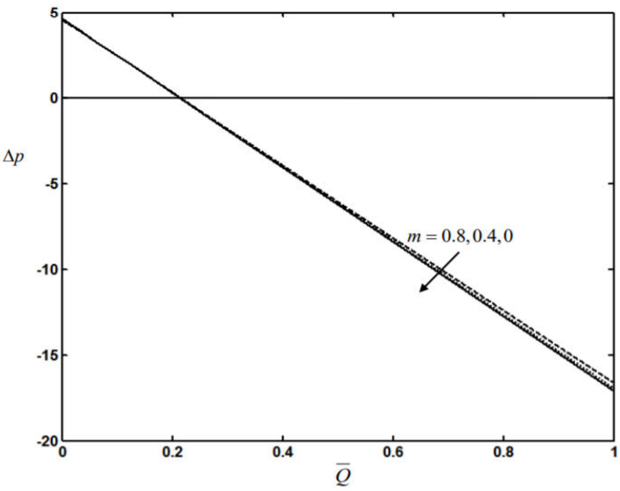

Figure-3. The Δp with $\overline{Q}$ for different values of Hall parameter m with $Da = 0.1$, $\text{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $M = 1$.

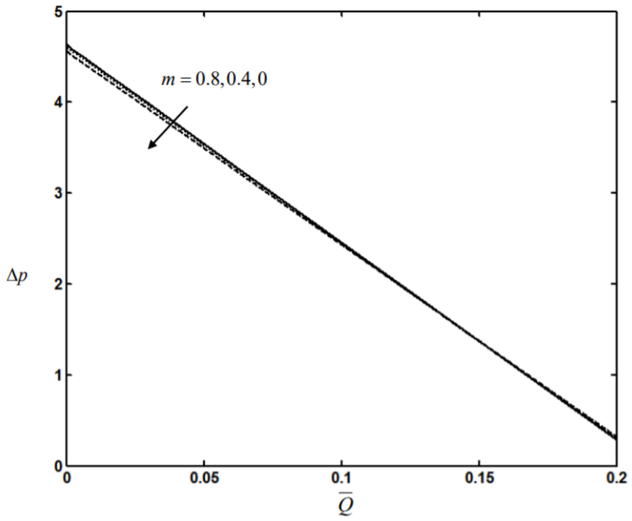

Figure-3(i). Enlargement of Figure-3.

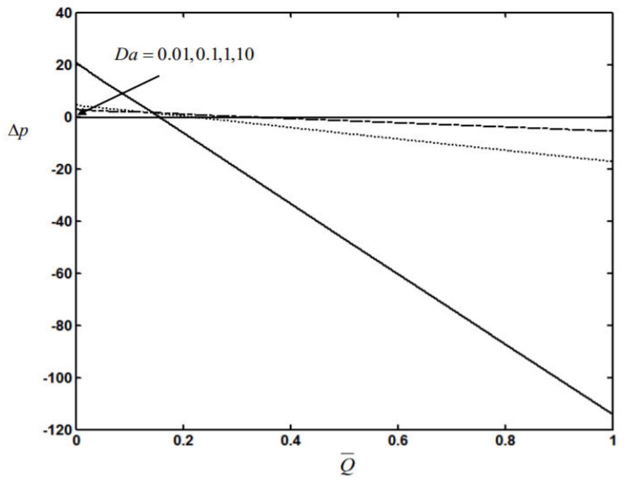

Figure-4. The Δp with $\overline{Q}$ for different values of Darcy number Da with $m = 0.2$, $\mathrm{Re} = 0.5$, $Fr = 2$, $\phi = 0.5$ and $M = 1$.

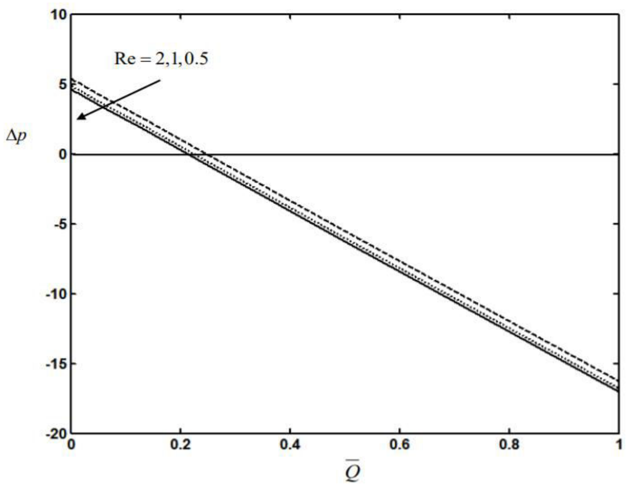

Figure-5. The Δp with rate $\overline{Q}$ for altered values Re with $m = 0.2$, $Da = 0.1$, $Fr = 2$, $\phi = 0.5$ and $M = 1$.

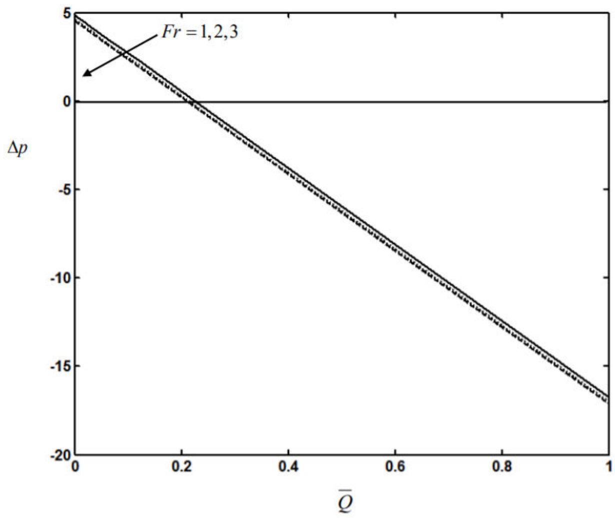

Figure-6. The Δp with $\overline{Q}$ for dissimilar values of Fr with $m = 0.2$, $\mathrm{Re} = 0.5$, $Da = 0.1$, $\phi = 0.5$ and $M = 1$.

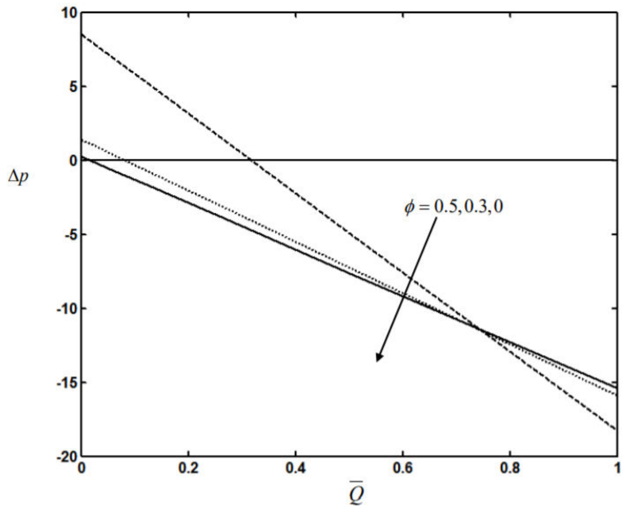

Figure-7. The Δp with $\overline{Q}$ for distinctive values of ϕ with $\mathrm{Re} = 0.5$, $Fr = 2$, $M = 1$ and $m = 0.2$.

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