



# DETERMINATION OF THE AVERAGE RESERVOIR PRESSURE FOR HORIZONTAL WELLS FROM BUILDUP, DRAWDOWN AND MULTI-RATE TESTS IN HOMOGENEOUS AND NATURALLY FRACTURED FORMATIONS BY THE TDS TECHNIQUE

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## ABSTRACT

The importance of the average reservoir pressure in all phases of the hydrocarbon industry implies to conduct continues research for better calculations of this property. In this paper the pseudosteady-state pressure solution in hydraulically fractured wells combined with a better determination of the effective wellbore radius is used to provide accurate estimations of the average reservoir pressure. These solutions initially presented for homogeneous formations are also extended to naturally fractured reservoirs and expressions for reservoir shape factors and average reservoir pressure are introduced for flow, pressure buildup and multi-rate tests for either liquids or gas reservoirs. The expressions are successfully tested with synthetic examples and compared to calculations from material balance given absolute deviation errors lower than 0.25 %.

**Keywords:** pseudosteady state, TDS technique, bounded reservoir, anisotropic reservoir, shape factor, pressure derivative, superposition, multi-rate tests.

## 1. INTRODUCTION

Recent research on the estimation of the average reservoir pressure was presented by Chacón *et al.* (2004), who extended the *TDS* Technique, for vertical wells, vertical fractured wells and horizontal wells. The last one used the the late time pressure solution of a hydraulically fractured well. Another extension of the *TDS* Technique, Tiab (1993), to estimate the average reservoir pressure in naturally fractured reservoirs was given by Molina *et al.* (2004). Escobar, Ibagón and Montealegre-M. (2007) presented the *TDS* methodology to estimate the average reservoir pressure for vertical wells in either homogeneous or heterogeneous formation under multi-rate testing. Recently, Escobar, Palomino and Suescun-Diaz (2020) presented a *TDS* methodology, following Agarwal's (2010) idea, to estimate the average pressure for drawdown tests conducted in horizontal wells but treating the well as it were a fractured well since this and horizontal well behave similarly, mathematically speaking, as performed by Chacon et al (2004) and Escobar *et al.* (2011).

Agarwal (2010) performed a mathematical treatment of pressure drawdown and material balance equations to determine the average reservoir pressure from flow tests. A similar idea was employed by Mohammed, Enty, and Amarfio (2014) to provide and estimation of the average reservoir pressure in constant-rate drawdown. Escobar, Palomino and Jongkittinarukorn (2019) used Agarwal's idea to extend the *TDS* Technique to obtain equations for the estimation of the average reservoir pressure and shape factors in vertical wells in homogeneous and heterogenous formations and hydraulically fractured wells in homogeneous reservoirs.

In this work the late time pseudosteady-state pressure solutions presented by Ozkan (1988) for

cylindrical and rectangular formations drained by horizontal wells were used to find more accurate solutions to find the average reservoir pressure in drawdown, buildup, multi-rate tests for hydrocarbon reservoirs. Excellent match was obtained from the provided solutions as compared to results obtained from material balance.

## 2. MATHEMATICAL DEVELOPMENT - BUILDUP TESTING

### 2.1 Homogenous Anisotropic Reservoirs

The dimensionless pressure equation for both a horizontal and a vertical well, respectively:

$$P_D = \frac{\bar{k}L_w(P_i - P)}{141.2q\mu B} \quad (1)$$

The dimensionless time based upon wellbore length, and reservoir drainage area are given as:

$$t_D = \frac{0.0002637\bar{k}t}{\phi\mu c_t L_w^2} \quad (2)$$

$$t_{DA} = \frac{0.0002637\bar{k}t}{\phi\mu c_t A} \quad (3)$$

Raghavan (1993) stated that material balance for a slightly compressible fluid in bounded reservoirs can be expressed as:

$$\bar{P}_D(t_{DA}) = 2\pi t_{DA} \quad (4)$$



### 2.1.1 Cylindrical reservoirs

The late pseudosteady-state pressure behavior for a horizontal well in an anisotropic formation was presented by Ozkan (1988).

$$P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{4Ae^{2[1+\sigma(x_D,0)+\delta(x_D,0,r_D)+F+Fb]}}{e^{\gamma} C_A L_w^2} \right\} \quad (5)$$

Which can be expressed as:

$$P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (6)$$

Which pressure derivative is given by:

$$t_{DA} * P_D' = P_{Dmb}(t_{DA}) = 2\pi t_{DA} \quad (7)$$

Division of Equation(6) by Equation (7) will provide:

$$\frac{P_D}{(t_{DA} * P_D')} = 1 + \frac{1}{4\pi t_{DA}} \ln \left\{ \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (8)$$

Substituting the dimensionless quantities given by Equations (1) and (3) into Equation (8), and solving for the shape factor,  $C_A$ , will result:

$$C_A = \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp \left[ \frac{\bar{k}t_{pss}}{301.77\phi\mu c_i A} \left( \frac{(\Delta P)_{pss}}{(t * \Delta P')_{pss}} - 1 \right) \right]} \quad (9)$$

Using the concept given by Equation (4) in Equation (8), this becomes:

$$\frac{P_D}{(t_{DA} * P_D')} = 1 + \frac{1}{2\bar{P}_D} \ln \left\{ \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (10)$$

Substituting Equation (1) and its derivative in Equation (10) and solving for the average reservoir pressure,

$$\bar{P} = P_i - \frac{70.6q\mu B}{\bar{k}L_w} \left( \frac{(t * \Delta P')_{pss}}{(\Delta P)_{pss} - (t * \Delta P')_{pss}} \right) \times \ln \left\{ \frac{8.9834Ae^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (11)$$

### 2.1.2 Rectangular systems

Ozkan (1988) also introduced pseudosteady-state pressure behavior for a horizontal well in an anisotropic rectangular formation,

$$P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{4A}{e^{\gamma} C_A L_w^2} \right\} \quad (12)$$

which can be rewritten as,

$$P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{2.2458A}{C_A L_w^2} \right\} \quad (13)$$

Equation (7) is also the derivative of Equation (13). Then, dividing Equation (13) by Equation (7), it yields,

$$\frac{P_D}{(t_{DA} * P_D')} = 1 + \frac{1}{4\pi t_{DA}} \ln \left\{ \frac{2.2458A}{C_A L_w^2} \right\} \quad (14)$$

After plugging Equations (1) ad (3) in the above expression and, then, solving for the shape factor,

$$C_A = \frac{2.2458A}{L_w^2 \exp \left[ \frac{\bar{k}t_{pss}}{301.77\phi\mu c_i A} \left( \frac{(\Delta P)_{pss}}{(t * \Delta P')_{pss}} - 1 \right) \right]} \quad (15)$$

Again, applying the concept of Equation (4) in Equation (14),

$$\frac{P_D}{(t_{DA} * P_D')} = 1 + \frac{1}{2\bar{P}_D} \ln \left\{ \frac{2.2458A}{C_A L_w^2} \right\} \quad (16)$$

Substitution of Equation (1) and its derivative in Equation (14) leads to solve for the average reservoir pressure,

$$\bar{P} = P_i - \frac{70.6q\mu B}{\bar{k}L_w} \left( \frac{(t * \Delta P')_{pss}}{(\Delta P)_{pss} - (t * \Delta P')_{pss}} \right) \ln \left\{ \frac{2.2458A}{C_A L_w^2} \right\} \quad (17)$$

### 2.2 Other Parameters

Tiab (1994) found an expression to estimate the drainage area from the intersection point of the radial and pseudosteady-state flow regimes,  $t_{rpi}$ , by using:

$$A = \frac{\bar{k}t_{rpi}}{301.77\phi\mu c_i} \quad (18)$$

where,

$$\bar{k} = \sqrt{k_x k_y} \quad (19)$$

The different skin factors and other reservoir parameters were found by Engler and Tiab (1996a, 1996b), as:

$$s_m = \frac{1}{2} \left[ \frac{\Delta P_{er}}{(t * \Delta P')_{er}} - \ln \left( \frac{\sqrt{k_y k_z} t_{er}}{\phi\mu c_i r_w^2} \right) + 7.43 \right] \quad (20)$$



$$s_m + s_z = \frac{L_w}{2h_z} \sqrt{\frac{k_z}{k_y}} \left[ \frac{\Delta P_{pr}}{(t^* \Delta P')_{pr}} - \ln \left( \frac{k_x t_{pr}}{\phi \mu c_l L_w^2} \right) + 4.659 \right] \quad (21)$$

$$s_m + s_z + s_x = \frac{0.029 L_w}{h_x h_z} \sqrt{\frac{k_z t_{ll}}{\phi \mu c_l}} \left[ \frac{\Delta P_{ll}}{(t^* \Delta P')_{ll}} - 2 \right] \quad (22)$$

Martinez, Escobar and Bonilla (2012) presented an improved version of the skin factor due to elliptical flow,

$$s_{Ell} = \left[ \frac{\Delta P_{Ell}}{(t^* \Delta P')_{Ell}} - \frac{1}{0.36} \right] \frac{1}{25.424} \left( \frac{k_x t_{Ell}}{\phi \mu c_l L_w^2} \right)^{0.36} \quad (23)$$

The bounded skin factor,  $s_b$ , can be estimated by Equation (2.7.56) from Ozkan (1988) which involves the use of Bessel functions and integrals making the calculation somehow impractical; therefore,  $s_b$  can be neglected in Equations (9) and (11) and use the elliptical flow regime skin factor given by Equation (23).

### 2.3 Naturally Fractured Reservoirs

Equation (3) can be defined as:

$$t_{DA} = \frac{0.0002637 \bar{k} t}{\left[ (\phi c_l)_f + (\phi c_l)_m \right] \mu A} \quad (24)$$

The dimensionless storativity ratio introduced by Warren and Root (1963) is given by:

$$\omega = \frac{(\phi c_l)_f}{(\phi c_l)_f + (\phi c_l)_m} \quad (25)$$

Multiplying and dividing Equation (24) by  $(\phi c_l)_f$ , it results,

$$t_{DA} = \frac{0.0002637 \bar{k} t \omega}{(\phi c_l)_f \mu A} \quad (26)$$

Measuring the  $(\phi c_l)_f$  product is not practical. Therefore, Tiab, Igboykoyi, and Restrepo (2007) found,

$$(\phi c_l)_f = (\phi c_l)_m \left( \frac{\omega}{1-\omega} \right) \quad (27)$$

and,

$$(\phi c_l)_{f+m} = (\phi c_l)_m \left( 1 + \frac{\omega}{1-\omega} \right) \quad (28)$$

With this manipulation, Equation (9) and (15) become, respectively,

$$C_A = \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp \left[ \frac{\omega \bar{k} t_{pss}}{301.77 (\phi c_l)_f \mu A} \left( \frac{(\Delta P)_{pss}}{(t^* \Delta P')_{pss}} - 1 \right) \right]} \quad (29)$$

$$C_A = \frac{2.2458 A}{L_w^2 \exp \left[ \frac{\omega \bar{k} t_{pss}}{301.77 (\phi c_l)_f \mu A} \left( \frac{(\Delta P)_{pss}}{(t^* \Delta P')_{pss}} - 1 \right) \right]} \quad (30)$$

Notice that Equations (11) and (17) are unaffected since the new dimensionless time equation is absent from Equations (10) and (16). Therefore, Equations (11) and (17) can be used to estimate the average reservoir pressure once the shape factor is estimated, now, with Equations (29) and (30) depending upon a cylindrical or rectangular system is dealt with.

### 2.4 Multi-Rate Testing

Onur *et al.* (1988) presented the normalized pressure approach concept. Also, Earlougher (1977) presented the variable-rate governing equation:

$$\Delta P_q = \frac{162.6 \mu B}{kh} \left[ X_n + \log \frac{k}{\phi \mu c_l r_w^2} - 3.23 + 0.87s \right] \quad (31)$$

where,

$$\Delta P_q = \frac{P_i - P_{wf}(t)}{q_n} \quad (32)$$

$$X_n = \sum_{i=1}^n \left( \frac{q_i - q_{i-1}}{q_n} \right) \log(t - t_{i-1}) \quad (33)$$

The equivalent time concept applied to the *TDS* technique to multi-rate and variable injection tests were introduced by Mongi and Tiab (2000) and Hachlaf *et al.* (2002):

$$t_{eq} = \prod_{i=1}^n (t_n - t_{n-1})^{\left( \frac{q_i - q_{i-1}}{q_n} \right)} = 10^{X_n} \quad (34)$$

where;

$$t_n = t_{n-1} + \Delta t \quad (35)$$

A new redefinition of the dimensionless pressure for a horizontal well:

$$P_{Dq} = \frac{\bar{k} L_w}{141.2 \mu B} \left( \frac{P_i - P_{wf}(t)}{q_n} \right) \quad (36)$$

and the dimensionless times are now expressed as:



$$t_{D_{eq}} = \frac{0.0002637 \bar{k} t_{eq}}{\phi \mu c_i L_w^2} \quad (37)$$

$$t_{D_{A_{eq}}} = \frac{0.0002637 \bar{k} t_{eq}}{\phi \mu c_i A} \quad (38)$$

A similar procedure as the one performed for the constant-rate case is followed here to obtain the average pressure equation for a horizontal well producing at a continuously changing flow rate is given by:

$$\bar{P} = P_i - \frac{70.6 q_n \mu B}{\bar{k} L_w} \left( \frac{(t^* \Delta P')_{pss}}{(\Delta P_q)_{pss} - (t_{eq}^* \Delta P_q')_{pss}} \right) \times \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (39)$$

$$C_A = \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp \left[ \frac{\bar{k} (t_{eq})_{pss}}{301.77 \phi \mu c_i A} \left( \frac{(\Delta P_q)_{pss}}{(t_{eq}^* \Delta P_q')_{pss}} - 1 \right) \right]} \quad (40)$$

$$C_A = \frac{2.2458 A}{L_w^2 \exp \left[ \frac{\bar{k} (t_{eq})_{pss}}{301.77 \phi \mu c_i A} \left( \frac{(\Delta P_q)_{pss}}{(t_{eq}^* \Delta P_q')_{pss}} - 1 \right) \right]} \quad (41)$$

$$\bar{P} = P_i - \frac{70.6 q_n \mu B}{\bar{k} L_w} \left( \frac{(t_{eq}^* \Delta P_q')_{pss}}{(\Delta P_q)_{pss} - (t_{eq}^* \Delta P_q')_{pss}} \right) \ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\} \quad (42)$$

Equations (39) and (40) are given for cylindrical reservoirs and Equations (41) and (42) for a rectangular-shaped reservoir. By the same token, for naturally fractured reservoirs for the mentioned systems, the estimation of the shape factor is given by:

$$C_A = \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp \left[ \frac{\omega \bar{k} (t_{eq})_{pss}}{301.77 (\phi c_i)_f \mu A} \left( \frac{(\Delta P_q)_{pss}}{(t_{eq}^* \Delta P_q')_{pss}} - 1 \right) \right]} \quad (43)$$

$$C_A = \frac{2.2458 A}{L_w^2 \exp \left[ \frac{\omega \bar{k} (t_{eq})_{pss}}{301.77 (\phi c_i)_f \mu A} \left( \frac{(\Delta P_q)_{pss}}{(t_{eq}^* \Delta P_q')_{pss}} - 1 \right) \right]} \quad (44)$$

## 2.5 GAS WELLS

The dimensionless pseudopressure derivative for gas flow is:

$$m(P)_D = \frac{\bar{k} L_w [m(P_i) - m(P)]}{1422.52 q_g T} \quad (45)$$

With these new dimensionless variables, Equations (11), (17), (39) and (42) become, respectively,

$$m(\bar{P}) = m(P_i) - \frac{711.28 q \mu B}{\bar{k} L_w} \left( \frac{(t^* \Delta P')_{pss}}{(\Delta P)_{pss} - (t^* \Delta P')_{pss}} \right) \times \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (46)$$

$$m(\bar{P}) = m(P_i) - \frac{70.6 q \mu B}{\bar{k} L_w} \times \left( \frac{(t^* \Delta P')_{pss}}{(\Delta P)_{pss} - (t^* \Delta P')_{pss}} \right) \ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\} \quad (47)$$

$$m(\bar{P}) = m(P_i) - \frac{711.28 q_n \mu B}{\bar{k} L_w} \times \left( \frac{(t^* \Delta P')_{pss}}{(\Delta P_q)_{pss} - (t_{eq}^* \Delta P_q')_{pss}} \right) \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (48)$$

$$m(\bar{P}) = m(P_i) - \frac{711.28 q_n \mu B}{\bar{k} L_w} \times \left( \frac{(t_{eq}^* \Delta P_q')_{pss}}{(\Delta P_q)_{pss} - (t_{eq}^* \Delta P_q')_{pss}} \right) \ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\} \quad (49)$$

Care must be taken with Equations (9), (15), (29), (30), (40), (41), (43) and (44) to change the pressure derivative ratio by the pseudopressure derivative ratio. For example, for Equation (9),

$$C_A = \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{L_w^2 \exp \left[ \frac{\bar{k} t_{pss}}{301.77 \phi \mu c_i A} \left( \frac{[\Delta m(P)]_{pss}}{[t^* \Delta m(P)]_{pss}} - 1 \right) \right]} \quad (50)$$

## 3. MATHEMATICAL DEVELOPMENT - DRAWDOWN TESTING

### 3.1 Homogeneous Reservoirs

Escobar, Palomino and Suescun-Diaz (2020) took the dimensionless pressure equation for both a horizontal and a vertical well, respectively:

$$P_D(t_{DA}) = \frac{\bar{k} h_x (P_i - P_{wf})}{141.2 q B \mu} \quad (51)$$

$$P_{Dmb}(t_{DA}) = \frac{\bar{k} h_x (P_i - \bar{P})}{141.2 q B \mu} \quad (52)$$

$$\bar{P}_D(t_{DA}) = \frac{\bar{k} h_x (\bar{P} - P_{wf})}{141.2 q B \mu} \quad (53)$$



Following the idea of Agarwal (2010) to solve for the dimensionless average reservoir pressure from Equation (7), using the arithmetic derivative, leads to:

$$\bar{P}_D(t_{DA}) = P_D(t_{DA}) - P_{Dmb}(t_{DA}) \quad (54)$$

Combination of Equation (6) - cylindrical system-, (7) and (53) yields:

$$P_D = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} - 2\pi t_{DA} \quad (55)$$

Dividing Equation (55) by Equation (7) gives,

$$\bar{P}_D(t_{DA}) = 2\pi t_{DA} + \frac{1}{2} \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} - 2\pi t_{DA} \quad (56)$$

$$\frac{\bar{P}_D(t_{DA})}{t_D * P_D'} = \frac{1}{4\pi t_{DA}} \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (57)$$

Combination of Equations (53), (57), the derivative of Equation (1) and (3) and solving for the average reservoir pressure gives:

$$\bar{P} = P_{wf} + \frac{301.77(t * \Delta P')_{P_{wf}} \phi \mu c_t A}{\bar{k} t_{P_{wf}}} \times \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (58)$$

As stated by Agarwal (2010), the well flowing pressure,  $P_{wf}$ , corresponds to the time,  $t_{P_{wf}}$ , at which the Cartesian derivative becomes flat. Also,  $(t * \Delta P')_{P_{wf}}$  corresponds the pressure derivative at  $t_{P_{wf}}$ .

By the same token for the rectangular system, Equation (13), it results:

$$\bar{P} = P_{wf} + \frac{301.77(t * \Delta P')_{P_{wf}} \phi \mu c_t A}{\bar{k} t_{P_{wf}}} \ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\} \quad (59)$$

### 3.2 Naturally fractured Reservoirs

For the cylindrical case, combination of Equations (52), (56), the derivative of Equation (1) and (25) and solving for the average reservoir pressure gives:

$$\bar{P} = P_{wf} + \frac{301.77(t * \Delta P')_{P_{wf}} \mu(\phi c_t)_f A}{\omega \bar{k} t_{P_{wf}}} \times \ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\} \quad (60)$$

Following the same manipulations for the cylindrical systems, Equation (13), it gives:

$$\bar{P} = P_{wf} + \frac{301.77(t * \Delta P')_{P_{wf}} \mu(\phi c_t)_f A}{\omega \bar{k} t_{P_{wf}}} \ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\} \quad (61)$$

For gas wells, Equations (59) and (60) will become,

$$m(\bar{P}) = m(P_{wf}) + \frac{301.77[t * \Delta m(P')]_{P_{wf}} \mu(\phi c_t)_f A}{\omega \bar{k} t_{P_{wf}}} \times \quad (62)$$

$$\ln \left\{ \frac{8.9834 A e^{2[1+s_z+s_x+s_m+s_b]}}{C_A L_w^2} \right\}$$

$$m(\bar{P}) = m(P_{wf}) + \frac{301.77[t * \Delta m(P')]_{P_{wf}} \mu(\phi c_t)_f A}{\omega \bar{k} t_{P_{wf}}} \quad (63)$$

$$\ln \left\{ \frac{2.2458 A}{C_A L_w^2} \right\}$$

The shape factors are found using the expressions for pressure buildup tests.

**Table-1.** Fluid, reservoir and well data for simulated examples.

Parameter	Example1	Example2
$\bar{k}$ , md	220	700
$k_z$ , md	44	70
$\phi$ , %	20	
$c_r$ , 1/psi	$3 \times 10^{-6}$	
$h$ , ft	180	100
$Z_w$ , ft	90	50
$r_w$ , ft	0.35	0.3
$q$ , bbl/D	300	550
$B$ , rb/STB	1.32	1.2
$\mu$ , cp	0.8	3
$P_i$ , psi	2500	2750
$L_w$ , ft	1500	2000
$h_x$ , ft		10000
$A$ , Ac	7212.1	4591.4
$(\phi c_t)_f$ , 1/psi		$1 \times 10^{-5}$
$(\phi c_t)_{f+m}$ , 1/psi		$1 \times 10^{-4}$
$\omega$		0.1
$\lambda$		$1 \times 10^{-6}$



4. SIMULATED EXAMPLES

4.1 Example 1

A synthetic test was generated for a horizontal well in a homogeneous reservoir using information of second column in Table-1. The reservoir was assumed to be cylindrical with a radius of 10000 ft. Pressure and pressure derivative data are reported in Figure-1.

**Solution.** The below information was read from Figure-1.

$$\begin{aligned}
 t_{Ell} &= 1.01 \text{ psi} & (\Delta P)_{Ell} &= 2.098 \text{ psi} & (t^* \Delta P^*)_{Ell} &= 10.78 \text{ psi} \\
 t_{pr} &= 50.61 \text{ psi} & (\Delta P)_{pr} &= 3.94 \text{ psi} & (t^* \Delta P^*)_{pr} &= 0.554 \text{ psi} \\
 t_{pss} &= 350 \text{ psi} & (\Delta P)_{pss} &= 5.16 \text{ psi} & (t^* \Delta P^*)_{pss} &= 0.984 \text{ psi}
 \end{aligned}$$

Use of Equation (23) allows to find  $s_{ELL} = 1.09$  and Equation (21) allows to find  $s_m + s_z = 4.71$  giving a total skin factor of 5.8. Use of Equation (9) leads to find the horizontal well shape factor  $C_A = 744280.9$ . The average reservoir pressure was found to be 2490.84 psi while a value of 2497.27 is found from a commercial software using material balance. Notice that for this case  $C_A = 31.62$  which corresponds to vertical well in a circular reservoir.

4.2 Example 2

A simulated horizontal well pressure test was generated for a heterogeneous (naturally fractured) reservoir using information of third column of Table-1. The shape of the reservoir was taken as rectangular with 10000 ft by 20000 ft. Pressure and pressure derivative data are reported in Figure-2.

**Solution.** The characteristic points were read from Figure-2.

$$t_{Pwf} = 825.1 \text{ psi} \quad (\Delta P)_{Pwf} = 17.81 \text{ psi} \quad (t^* \Delta P^*)_{Pwf} = 6.54 \text{ psi}$$

which can serve as a point on the pseudosteady-state period. Then,

$$t_{pss} = 825.1 \text{ psi} \quad (\Delta P)_{pss} = 17.81 \text{ psi} \quad (t^* \Delta P^*)_{pss} = 6.54 \text{ psi}$$

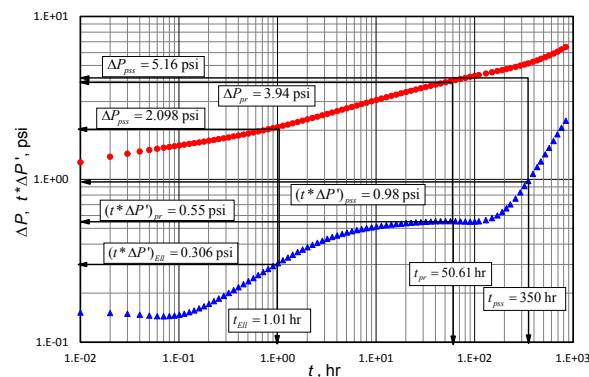


Figure-1. Pressure and pressure derivative versus time log-log plot for synthetic example 1.

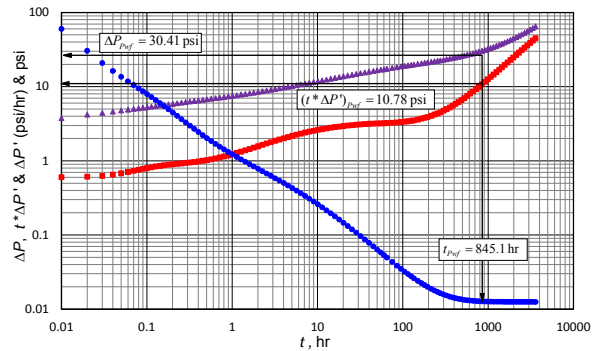


Figure-2. Pressure, pressure derivative and arithmetic pressure derivative versus time log-log plot for synthetic example 2.

The well-flowing pressure for this example is  $P_i - \Delta P_{wf} = 2750 - 17.81 = 2732.2$  psi. Equation (15) allows to find  $C_A = 106.45$  which is used in Equation (59) to provide an average reservoir pressure of 2743.13 psi. Also, using a commercial software with a  $C_A$  value of 21.83 which corresponds to a vertical well in a rectangular reservoir, a value of average reservoir pressure of 2742.86 psi was found by material balance.

5. COMMENTS ON THE RESULTS

One buildup and one drawdown tests were worked to demonstrate the accuracy of the proposed equations since the idea is not to demonstrate the applicability of the TDS Technique because it has been already demonstrated as presented by Escobar, Jongkittnarukorn, and Hernandez (2018) and later compiled by Escobar (2019) into a book. For the given case, in the first example an absolute deviation of 0.25 % was obtained as compared to material balance from a commercial simulator and 0.016 % deviation error was found for the second example -a naturally fractured reservoir- demonstrating the accuracy of the proposed equations.

6. CONCLUSIONS

New solutions to find the average reservoir pressure for pressure drawdown, pressure buildup and multirate hydrocarbon reservoirs in either homogeneous and naturally fractured reservoirs are provided. The equations are tested with synthetical examples finding absolute deviation errors less than 0.25 % compared to results from material balance provided by commercial specialized well testing software.

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## NOMENCLATURE

$A$	Drainage area, ft <sup>2</sup>
$B$	Oil volume factor, rb/STB
$C_A$	Dietz's shape factor
$c_t$	Total compressibility, 1/psi
$h_z$	Formation thickness, ft
$h_x$	Reservoir length along $x$ -direction, ft
$\bar{k}$	Horizontal permeability (Eq. 19), md
$k_x$	Reservoir permeability in $x$ -direction, md
$k_y$	Reservoir permeability in $y$ -direction, md
$k_z$	Reservoir permeability in vertical direction,
$L_w$	Effective horizontal well length, ft
$m(P)$	Pseudopressure function, psi <sup>2</sup> /cp
$P$	Pressure, psi
$\bar{P}$	Average reservoir pressure, psi
$P_D$	Dimensionless pressure
$P_i$	Initial pressure, psi
$P_{wf}$	Well-flowing pressure, psi
$q$	Oil flow rate, BPD
$q_g$	Gas flow rate, MSCF/D
$r_w$	Wellbore radius, ft
$s_b$	Reservoir boundary skin factor, $F_b$ Eq. (5)
$s_m$	Mechanical or infinite skin factor, $F$ Eq. (5)
$s_z$	Vertical skin factor, $\sigma$ in Eq. (5)
$s_x$	$x$ -direction skin factor due to partial penetration effects in the $x$ -direction parallel to the wellbore, $\delta$ in Eq. (5)
$t$	Time, hrs
$t_{eq}$	Equivalent time, hrs
$t^* \Delta P'$	Pressure derivative, psi
$t^* \Delta m(P)'$	Pseudopressure derivative, psi <sup>2</sup> /cp
$t_{eq}^* \Delta P_q'$	Normalized pressure derivative, psi/(STB/D)
$t_D$	Dimensionless time referred to wellbore radius
$t_{DA}$	Dimensionless time referred to reservoir area
$t_{rpi}$	Intercept of radial and pseudosteady state lines, hr
$t_{DA}^* P_D'$	Dimensionless pressure derivative based on reservoir area
$t_{DA}^* P_D'$	Dimensionless pressure derivative based on well radius
$T$	Reservoir temperature, °R
$x_D$	Dimensionless distance in the $x$ -direction
$x_e$	Reservoir length, md
$X_N$	Superposition time

## Subscripts

$D$	Dimensionless quantity
$Ell$	Elliptical
$er$	Early radial
$eq$	Equivalent
$f$	Formation or fracture
$i$	Initial conditions, intercept
$ll$	Late linear
$mb$	Material balance
$n$	Rate number
$pss, p$	Pseudosteady-state
$pr$	Pseudoradial
$P_{wf}$	At well-flowing pressure
$r$	Radial

## Greek

$\Delta$	Change, drop
$\Delta P_q$	Rate-normalized pressure drop, si/(STB/D)
$\phi$	Porosity, fraction
$\lambda$	Interporosity flow parameter
$\mu$	Viscosity, cp
$\omega$	Dimensionless storativity ratio