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ANALYSIS OF A SINGLE SERVER NON-PREEMPTIVE FUZZY PRIORITY QUEUE USING LR METHOD

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ABSTRACT

This paper investigates the performance measures for non-preemptive priority fuzzy queues. We analyse the characteristics of a single server in fuzzy environment with non-preemptive priority queueing model by LR method. We take both the arrival time and service time as fuzzy numbers. LR method has the advantage of being short and convenient compared to other methods such as alpha-cuts method. A numerical example is given to derive the performance measures of 2-priority queues.

Keywords: Non-Preemptive priority, LR method, triangular fuzzy number.

INTRODUCTION

In common we do not prefer to wait. Hence we use different models and methods to analyse such circumstances. Analysis of queues mathematically reduces the waiting time and waiting line. Queueing models are applied in several fields such as transportation engineering, service industry, production, communication systems, health care and information processing systems.

The waiting discipline in which customers are served according their order of arrival is normally found in queuing models, but still many real queueing systems follow priority discipline model. A priority mechanism is a method where different types of customers receive different performance level. Priority queues are applied in different field such as communication network, call centres and hospitals etc. Priority schemes are also known for their ease of implementation. Many queueing works analyse the priority queues. There are two possible segments in priority situation, the preemption and non-preemption. In the preemptive priority a high priority customer is allowed to receive the service instantly even though a lower priority customer is already in service. The service of low customer is stopped when higher class customer arrive and will be resumed from the point of pause, when all the queues of higher priority have been emptied. There is no interruption in non-preemptive priority queue. The non-preemptive priority queues are helpful in performance evaluation of computer systems and inventory controls. In this paper the queuing model considered has a non-preemptive priority service. The parameters in the priority queue may be fuzzy. The fuzzy queuing models are more useful and practical. Queuing models with fuzzy increases their application.

Bellman and Zadeh [1] presented the idea of fuzziness so that inexact information could be solved by decision making problems. The researchers like Li and Lee [11], Negi and Lee [14], Kaufmann [10], Chen [4, 5] have studied fuzzy queues. Fuzzy queues have been analysed using parametric linear programming approach by Kao *et al* [9]. F. Choobinesh and H. Li Zadeh [6] discussed on different methods for ordering fuzzy numbers. R. Jain [8] has discussed on decision making with fuzzy variables.

L.A Zadeh [18, 19] has presented the idea of fuzzy probabilities and the fuzzy probability Markov chains were also discussed. Buckley [2] studies multi-server queues with finite and infinite capacity queueing models where arrivals and departures follow possibilistic pattern. Chen [3] suggests a strategy of parametric programming in order to derive membership functions of the fuzzy queues. Further, the conversion of fuzzy queues to crisp queues has also been widely discussed in works and many methods and approaches have been used. M.J. Pardo and David de la Fuente [16] optimized a fuzzy priority discipline queuing model. B. Palpandi, and G. Geetharamani [15] computed performance measures of fuzzy non-preemptive priority queues by Robust ranking technique. W. Ritha and L. Robert [17] have analysed priority queuing discipline using fuzzy set theory. Non-preemptive priority fuzzy queues have been studied by Devaraj and Jayalakshmi [7] where fuzzy problem are reduced to crisp problem. J.P. Mukeba, R. Mabela and B. Ulungu [12, 13] derived the performance measures of fuzzy retrial queue using LR method. Here we study the non-preemptive priority queue and henceforth derive the performance measures of fuzzy non-preemptive priority queues using LR ranking technique.

Preliminaries

Definition

A fuzzy set is characterized by a membership function mapping elements of a domain space, or universe of discourse X to the unit interval [0, 1]. (i.e.) A = $\{(x,\mu_A(x));\ x \in X\}$. Here $\mu_A : X \rightarrow [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x $\in$ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition

Let E be a classical set or a universe. A fuzzy subset $\tilde{A}$(or a fuzzy set $\tilde{A}$) in E is defined by the function $\eta_{\tilde{A}}$, called membership function of $\tilde{A}$, from E to the real unit interval [0, 1]. In these conditions $\eta_{\tilde{A}}(a)$, is called the grade or the membership degree of $a, \forall a \in \tilde{A}$. For each $x \in E$, $\eta_{\tilde{A}}(x) = 1$, x is said mean value, modal value or mode of $\tilde{A}$.

Definition

Let $\tilde{A}$ be a fuzzy subset in the universe E. The alpha-cut $\tilde{A}_\alpha$, the support supp $(\tilde{A})$, the height hgt$(\tilde{A})$, and the core core$(\tilde{A})$of $\tilde{A}$, are crisp sets defined respectively as follows $\forall \alpha \in [0,1]$

$$\tilde{A}_\alpha = \{x \in E | \eta_{\tilde{A}}(x) \geq \alpha\}$$
$$\text{supp}(\tilde{A}) = \{x \in E | \eta_{\tilde{A}}(x) > 0\}$$
$$\text{hgt}(\tilde{A}) = max\{\eta_{\tilde{A}}(x) | x \in E\}$$
$$\text{core}(\tilde{A}) = \{x \in E | \eta_{\tilde{A}}(x) = 1\}$$

Definition

A fuzzy set $\tilde{A}$ is said normal if and only if hgt$(\tilde{A}) = 1$ and convex if and only if $\eta_{\tilde{A}}(\lambda x + (1-\lambda y)) \geq \min\{\eta_{\tilde{A}}(x), \eta_{\tilde{A}}(y)\}$ $\forall x, y \in \tilde{A}, \forall \alpha \in [0,1]$

Definition

A fuzzy number $\tilde{A}$ is said positive if and only if $\forall x < 0, \eta_{\tilde{A}}(x) = 0$ and negative if and only $\forall x > 0, \eta_{\tilde{A}}(x) = 0$.

Definition

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers. $\tilde{A} < \tilde{B}$ if and only if
$\forall x \in supp(\tilde{A}), \forall y \in supp(\tilde{B}), x < y$. In other words, $\tilde{A} < \tilde{B} \Leftrightarrow sup\{supp(\tilde{A})\} < inf\{supp(\tilde{B})\}$

α-cut of a fuzzy number

The α-cut of a fuzzy number A(x) is defined as $A(\alpha) = \{x: \mu(x) \geq \alpha, \alpha \in [0,1]\}$

Addition of two Triangular fuzzy numbers can be performed as

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

Triangular fuzzy number

For a Triangular number $A(x)$, it can be represented by $A(\text{a, b, c,}; 1)$ with membership function μ(x) given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise \end{cases}$$

Such triangular fuzzy number is often noted as $\tilde{A} = (a, b, c)$. The LR representation can be written as $\tilde{A} = (a, b, c) = \langle b, b-a, c-b \rangle_{LR}$ for $L(x) = R(x) = max(0, 1-x)$

L-R fuzzy number

A fuzzy number $\widetilde{M}$ is said L-R fuzzy number if and only if there exists three numbers $m, a > 0, b > 0$ and two positive, continuous and decreasing functions L and R from real number to [0,1], such that

$$L(0) = R(0) = 1$$
$$L(1) = 0, L(x) > 0, \lim_{x \to \infty} L(x) = 0$$
$$R(1) = 0, R(x) > 0, \lim_{x \to \infty} R(x) = 0$$

$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\dfrac{m-x}{a}\right) if x \epsilon [m-a, m] \\ R\left(\dfrac{x-m}{b}\right) if x \epsilon [m, m+b] \\ 0 \quad otherwise \end{cases}$$

The L-R representation of the fuzzy number $\widetilde{M}$ is $\widetilde{M} = \langle \text{m, a, b} \rangle_{\text{LR.}}$ m is called the mean value or modal value of $\widetilde{M}$, a *and* b are called the left spread and right spread of $\widetilde{M}$. Conventionally, $\widetilde{M} = \langle \text{m}, 0, 0 \rangle_{\text{LR.}}$ is the ordinary real number m, called fuzzy singleton.

$$supp(\widetilde{M}) =]m-a, m] \cup [m, m+b[=]m-a, m+b[$$

Arithmetic of L-R fuzzy numbers

If there are fuzzy numbers of the same type $\widetilde{M} = \langle \text{m, a, b} \rangle_{\text{LR.}}$ and $\widetilde{N} = \langle \text{n, c, d} \rangle_{\text{LR.}}$ then their sum and their difference are also L-R fuzzy numbers of the same type given by

$$\widetilde{M} + \widetilde{N} = \langle m+n, a+c, b+d \rangle_{LR}$$
$$\widetilde{M} - \widetilde{N} = \langle m-n, a+d, b+c \rangle_{LR}$$

The product of L-R fuzzy numbers $\widetilde{M} = \langle \text{m, a, b} \rangle_{\text{LR.}}$ and $\widetilde{N} = \langle \text{n, c, d} \rangle_{\text{LR.}}$ is given by

$$\widetilde{M}.\widetilde{N} \approx \langle m.n, mc + na - ac, md + nb + bd \rangle_{LR}$$

The quotient secant approximation of two L-R fuzzy numbers $\widetilde{M} = \langle \text{m, a, b} \rangle_{\text{LR.}}$ and $\widetilde{N} = \langle \text{n, c, d} \rangle_{\text{LR.}}$ is given by

$$\frac{\widetilde{M}}{\widetilde{N}} = \frac{\langle m, a, b \rangle_{LR}}{\langle n, c, d \rangle_{LR}} \approx \langle \frac{m}{n}, \frac{md}{n(n+d)} + \frac{a}{n} - \frac{ad}{n(n+d)}, \frac{mc}{n(n-c)} + \frac{b}{n} + \frac{bc}{n(n-c)} \rangle_{LR}$$

Non-Preemptive priority

Suppose that customers of the k^{th} priority arrive according to poisson process and service distribution for the k^{th} priority be exponentially distributed with mean $\frac{1}{\mu_k}$ unit that begins service and completes its service before another item is admitted, regardless of priority. We begin with $\rho_k = \frac{\lambda_k}{\mu_k}(1 \leq k \leq r), \sigma_k = \sum_{i=1}^{i=k} \rho_i (\sigma_0 = 0, \sigma_r = \rho)$. The system is stationary for $\rho < 1$. Without the loss of generality let us assume the performance measures for 2-priority queues. From the traditional queuing theory

$$W_{q,i} = \frac{\sum_{k=1}^{k=r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)}$$

$$L_q = \sum_{i=1}^{r} {L_q}^{(i)} = \sum_{i=1}^{r} \lambda_i(W_{q,i})$$

For the non-preemptive queues with 2-priority customers with equal service rates we have $\mu_1 = \mu_2 = \mu$, $\rho_1 = \frac{\lambda_1}{\mu_1}, \rho_2 = \frac{\lambda_2}{\mu_2}$.

Since $\rho = \rho_1 + \rho_2, \rho = \frac{\lambda}{\mu}, \lambda = \lambda_1 + \lambda_2, \sigma_k = \sum_{i=1}^{i=k} \rho_i, \sigma_0 = 0$

$$ {W_q}^{(1)} = \frac{\lambda}{\mu(\mu-\lambda_1)}$$

$$ {W_q}^{(2)} = \frac{\lambda}{(\mu-\lambda_1)(\mu-\lambda)}$$

$$ {L_q}^{(1)} = \frac{\lambda\lambda_1}{\mu(\mu-\lambda_1)}$$

$$ {L_q}^{(2)} = \frac{\lambda\lambda_2}{(\mu-\lambda_1)(\mu-\lambda)}$$

Characteristics of fuzzy non-preemptive priority queue

Let us consider all the arrival rates and service rate are LR fuzzy numbers noted by $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}$. Under these conditions the model becomes a fuzzy model denoted by FM/FM/1 where FM stands for a fuzzified exponential distribution. The crisp performance measures formula becomes

$$ \widetilde{W_q}^{(1)} = \frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda}_1)}$$

$$ \widetilde{W_q}^{(2)} = \frac{\tilde{\lambda}}{(\tilde{\mu}-\tilde{\lambda}_1)(\tilde{\mu}-\tilde{\lambda})}$$

$$ \tilde{L}_q^{(1)} = \frac{\tilde{\lambda}\tilde{\lambda}_1}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda}_1)}$$

$$ \tilde{L}_q^{(2)} = \frac{\tilde{\lambda}\tilde{\lambda}_2}{(\tilde{\mu}-\tilde{\lambda}_1)(\tilde{\mu}-\tilde{\lambda})}$$

Since $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}$ being fuzzy numbers we use fuzzy arithmetic. A fuzzy queue is stable if and only if $\tilde{\lambda}$ is less than $\tilde{\mu}$. In other words $sup\{supp(\tilde{\lambda})\} < inf\{supp(\tilde{\mu})\}$

Numerical example

Let the arrival rates with same service rates of triangular fuzzy numbers for first and second priority are represented as $\tilde{\lambda}_1 = [4,5,7], \tilde{\lambda}_2 = [8,10,12], \tilde{\mu} = [22,23,25], \tilde{\lambda} = [12,15,19]$

The membership function is given by

$$\mu_{\tilde{\lambda}_1} = \begin{cases} \frac{x-4}{1} & 4 \le x \le 5 \\ \frac{7-x}{2} & 5 \le x \le 7 \\ 0 & otherwise \end{cases}$$

Putting fuzzy parameters $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}, \tilde{\mu}$ in their L-R decomposition

$$\tilde{\lambda}_1 = \langle 5,1,2\rangle_{LR} \tilde{\lambda}_2 = \langle 10,2,2\rangle_{LR} \tilde{\lambda} = \langle 15,3,4\rangle_{LR} \tilde{\mu} = \langle 23,1,2\rangle_{LR}$$

Average waiting time of first priority customer in the queue is

$$\widetilde{W_q}^{(1)} = \frac{\langle 15,3,4\rangle_{LR}}{\langle 23,1,2\rangle_{LR}[\langle 23,1,2\rangle_{LR} - \langle 5,1,2\rangle_{LR}]}$$
$$= \frac{\langle 15,3,4\rangle_{LR}}{\langle 23,1,2\rangle_{LR}\langle 18,3,3\rangle_{LR}}$$
$$= \frac{\langle 15,3,4\rangle_{LR}}{\langle 414,84,111\rangle_{LR}}$$
$$= \langle 0.0362,0.0134,0.0213\rangle_{LR}$$

Average waiting time of second priority customer in the queue is

$$\widetilde{W_q}^{(2)}$$
$$= \frac{\langle 15,3,4\rangle_{LR}}{[\langle 23,1,2\rangle_{LR} - \langle 5,1,2\rangle_{LR}][\langle 23,1,2\rangle_{LR} - \langle 15,3,4\rangle_{LR}]}$$
$$= \frac{\langle 15,3,4\rangle_{LR}}{\langle 18,3,3\rangle_{LR}\langle 8,5,5\rangle_{LR}}$$
$$= \frac{\langle 15,3,4\rangle_{LR}}{\langle 144,99,129\rangle_{LR}}$$
$$= \langle 0.1042,0.0602,0.3181\rangle_{LR}$$

Average queue length of first priority is

$$\tilde{L}_q^{(1)} = \frac{\langle 15,3,4\rangle_{LR}\langle 5,1,2\rangle_{LR}}{\langle 23,1,2\rangle_{LR}[\langle 23,1,2\rangle_{LR} - \langle 5,1,2\rangle_{LR}]}$$
$$= \frac{\langle 75,27,58\rangle_{LR}}{\langle 23,1,2\rangle_{LR}\langle 18,3,3\rangle_{LR}}$$
$$= \frac{\langle 75,27,58\rangle_{LR}}{\langle 414,84,111\rangle_{LR}}$$
$$= \langle 0.18125,0.0897,0.2218\rangle_{LR}$$

Average queue length of second priority is

$$\tilde{L}_q^{(2)} = \frac{\langle 15,3,4\rangle_{LR}\langle 10,2,2\rangle_{LR}}{[\langle 23,1,2\rangle_{LR} - \langle 5,1,2\rangle_{LR}][\langle 23,1,2\rangle_{LR} - \langle 15,3,4\rangle_{LR}]}$$
$$= \frac{\langle 150,54,78\rangle_{LR}}{\langle 144,99,129\rangle_{LR}}$$
$$= \langle 1.0416,0.6900,4.025\rangle_{LR}$$

According to the definitions the modal values of $\widetilde{W_q}^{(1)}, \widetilde{W_q}^{(2)}, \tilde{L}_q^{(1)}, \tilde{L}_q^{(2)}$ are respectively $m_{\widetilde{W_q}^{(1)}} =$

$0.0362, m_{\widetilde{W}_q{}^{(2)}} = 0.1042, m_{\tilde{L}_q{}^{(1)}} = 0.18125, m_{\tilde{L}_q{}^{(2)}} = 1.0416$ and their supports are the following open intervals

$$supp\left(\widetilde{W}_q{}^{(1)}\right) =]0.0362 - 0.0134, 0.0362 + 0.0213[\;=]0.0228, 0.0575[$$

$$supp\left(\widetilde{W}_q{}^{(2)}\right) =]0.1042 - 0.0602, 0.1042 + 0.3181[\;=]0.044, 0.4223[$$

$$supp\left(\tilde{L}_q{}^{(1)}\right) =]0.18125 - 0.0897, 0.18125 + 0.2218[\;=]0.09155, 0.40305[$$

$$supp\left(\tilde{L}_q{}^{(2)}\right) =]1.0416 - 0.6900, 1.0416 + 4.025[\;=]0.3516, 5.0666[$$

RESULTS

The modal value $m_{\widetilde{W}_q{}^{(1)}} = 0.0362$ and the support $supp\left(\widetilde{W}_q{}^{(1)}\right) =]0.0228, 0.0575[$ indicate that the waiting time of first priority customer in the queue is approximately between 0.0228 and 0.0575. Its most possible value is 0.0362.

The modal value $m_{\widetilde{W}_q{}^{(2)}} = 0.1042$ and the support $supp\left(\widetilde{W}_q{}^{(2)}\right) =]0.044, 0.4223[$ indicate that the waiting time of second priority customer in the queue is approximately between 0.044 and 0.4223. Its most possible value is 0.1042.

The modal value $m_{\tilde{L}_q{}^{(1)}} = 0.18125$ and the support $supp\left(\tilde{L}_q{}^{(1)}\right) =]0.09155, 0.40305[$ indicate that the queue length of first priority is approximately between 0.09155 and 0.40305. Its most possible value is 0.18125.

The modal value $m_{\tilde{L}_q{}^{(2)}} = 1.0416$ and the support $supp\left(\tilde{L}_q{}^{(2)}\right) =]0.3516, 5.0666[$ indicate that the queue length of second priority is approximately between 0.3516 and 5.0666. Its most possible value is 1.0416

CONCLUSIONS

The fuzzy non-preemptive priority queues are represented more exactly and the analytic results are derived by LR method. Numerical examples for triangular fuzzy numbers are explained effectively to determine the validity of the suggested model. The method used here in the paper is more effective in finding the performance measures of fuzzy queues. The expected queue length and expected waiting time are derived and the results are given in LR representation. The future work can be done in examining the efficiency of this technique to other queueing models.

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