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SQUEEZE FILM BEHAVIOUR OF ROUGH ELLIPTICAL PLATES WITH MICRO POLAR FLUIDS

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ABSTRACT

The effect of surface roughness on squeeze film elliptical plates with micro-polar fluid is theoretically presented in the present analysis. Based upon the Christensen's stochastic model, stochastic Reynolds-type equation is derived. The closed form solutions are derived for squeeze film pressure, load carrying capacity and squeeze film time. The present analysis describes how the roughness influences the characteristics of the film squeezed between rough elliptical plates lubricated with micro polar fluid. The results yields increasing values of roughness parameter, coupling number and couple stress parameter leads to increase in pressure, load carrying capacity and squeeze film time.

Keywords: elliptical plates, micropolar fluid, reynolds equation, squeeze film, surface roughness.

1. INTRODUCTION

Fluids with microstructure are known as Micropolar fluids. The important characteristic of the Micropolar fluid is that it has increased viscosity, particularly for thin films. The Micropolar fluids belong to a class of fluids called polar fluids with non-symmetric stress tensor and it contains rigid spherical particle suspended in a viscous medium. This fluid shows two features such as Micro-rotational inertia and Micro-rotational effects. Verma *et al*[1] reported a study on porous inclined slider bearing having lubricated with a Micro-polar fluid and the effects of fluid material constants for the bearing features have been discussed. They have analyzed that the porous inclined slider bearing have more load carrying capacity with micro polar fluid than Newtonian fluid. Zaheeruddin and Isa [2] have examined the characteristics of Micropolar lubricant in the squeeze film porous spherical bearing. They have studied that the load carrying capacity decreases when the porosity parameter increases. Roopa Rajashekhar Anagod *et al* [3] have given a detailed analysis on squeeze film elliptical plates lubricated with non-Newtonian micro-polar fluid and discussed that load carrying capacity and squeeze film time decreases with increasing the film thickness. Theoretical investigations have been done by Hanumagowda *et al* [4] the effect of pressure-dependent viscosity (PDV) with micropolar fluid on squeeze film circular step plate. From the analysis they found that pressure, load carrying capacity and squeeze film time reduces with increasing the step distance. The effects of viscous dissipation and internal heat generation have been considered by Aissa and Mohammadin[5]. They studied that skin friction increases with increasing the microrotation parameter which effects the viscous field. Magdy A. Ezzat [6] discussed the free convection flow of conducting Micropolar fluid with thermal relaxation under the presence of heat sources and the results were used quite successfully in generalized thermo-elasticity theory.
Roughness is an element of surface texture. Roughness can be measured by the deviations in the direction of the real surface normal vector compared with its ideal form. If the calculated deviations are more, then the surface is considered as rough and if the deviations are less than it is considered as smooth. Syeda Tasneem Fathima *et al*[7] found that the features of squeeze film are more noticeable for rough porous rectangular plates with growing values of Hartman number. They have described that characteristics of squeeze film are more pronounced for rough rectangular with increasing values of magnetic parameter. Syeda Tasneem Fathima *et al*[8] have done an analysis on the effects of surface roughness on the MHD Reynolds equation on the basis of different dimensionless variables with conducting couple stress film on rough porous elliptical plates and rectangular plates. They have observed that the load carrying capacity gradually increases when the surface roughness and transverse magnetic field increases. Syeda Tasneem Fathima *et al* [9]have investigated that there was a notable effect on the bearing in presence of couple stress fluid and magnetic field. They found that surface roughness has significant impact on the characteristic of bearing. Lin *et al*[10] discussed the dynamic characteristics of parabolic-film slider bearings with Micropolar fluid and found that micropolar fluid enhances both load carrying capacities and dynamic coefficients. The hydromagnetic squeeze film performance have been analysed between two conducting rough porous elliptical plates by Vadher *et al* [11].These were indicated that the transverse roughness of the surface unfavorably affect the bearing system. A study on the effect of surface roughness in elastohydrodynamic lubrication by Bush *etal* [12] have presented that leading effects of roughness is insensitive to the accurate form of the autocorrelation function and parameter which describes the extent of roughness and nature of the anisotropy. The squeeze film bearings studied by Singh and Sinha [13] and Tsai-Wang Huang *et al.* [14] analyzed the steady load carrying capacity and the dynamic characteristics of inclined slider bearings.

In this paper, a theoretical representation of surface roughness on the squeeze film elliptical plates with micro-polar fluid is considered. The proposed work describes how the roughness can influence the characteristics of the squeeze film between rough elliptical plates lubricated with micro polar fluid.

2. MATHEMATICAL FORMULATION

The configuration of the squeeze film geometry under the consideration is depicted in Figure-1.Consider micropolar fluid flow through a pair of horizontal rough elliptical plates separated by a height *h*. The semi major and semi minor axis are a and b respectively. Let the lower elliptical plate be fixed and the upper elliptical plate will be moving with velocity $v = \frac{dh}{dt}$ towards the lower elliptical plate.

The following basic equations are derived from micro-polar fluid film theory

$$\left(\mu + \frac{\chi}{2}\right)\frac{\partial^2 w}{\partial y^2} - \chi\frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial y} = 0 \tag{2}$$

$$\left(\mu + \frac{\chi}{2}\right)\frac{\partial^2 w}{\partial y^2} - \chi\frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial z} = 0 \tag{3}$$

$$\gamma\frac{\partial^2 v_3}{\partial y^2} - \chi\frac{\partial u}{\partial y} - 2\chi v_3 = 0 \tag{4}$$

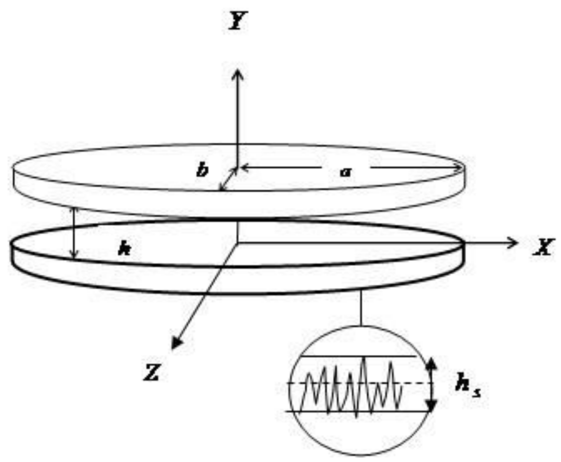

Figure-1. The physical representation of the bearing system.

$$\gamma\frac{\partial^2 v_1}{\partial y^2} - 2\chi v_1 + \chi\frac{\partial w}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

In the above equations u, v and w are the velocity components in *x*, y and z-directions.
Where p, μ and ηa are film pressure, the lubricant viscosity, material constant respectively

Corresponding boundary conditions are given by:
Since upper plate is moving along *Y*-direction.

$$u = 0, w = 0, v = \frac{\partial h}{\partial t} \tag{7}$$

$$v_1 = 0, v_3 = 0 \tag{8}$$

At the upper surface $y = 0$

$$u = 0, v = 0, w = 0 \tag{9}$$

$$v_1 = 0, v_3 = 0 \tag{10}$$

Solving equations (1)- (4) by using the boundary conditions as mentioned in the equations (7)- (10)we get

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\left[\frac{y^2}{2} - \frac{N^2 h}{m}\frac{(Coshmy-1)}{Sinhmh}\right] + \frac{D_1}{(1-N^2)}\left[y - \frac{N^2}{m}\left\{Sinhmy - (Coshmy-1)\frac{(Coshmh-1)}{Sinhmh}\right\}\right] \tag{11}$$

$$w = \frac{1}{\mu}\frac{\partial p}{\partial z}\left[\frac{y^2}{2} - \frac{N^2 h}{m}\frac{(Coshmy-1)}{Sinhmh}\right] + \frac{D_1}{(1-N^2)}\left[y - \frac{N^2}{m}\left\{Sinhmy - (Coshmy-1)\frac{(Coshmh-1)}{Sinhmh}\right\}\right] \tag{12}$$

where $m = \frac{N}{l}, l = \left(\frac{\gamma}{4\mu}\right)^{1/2}$

Substituting for w and u values in equation (6) and integrate under boundary conditions (7) to (9), we get

$$\frac{\partial}{\partial x}\left[\frac{dp}{dx}f(N,L,H)\right] + \frac{\partial}{\partial z}\left[\frac{dp}{dz}f(N,L,H)\right] = 12\mu\frac{dh}{dt} \tag{13}$$

To introduce the roughness model on elliptical plates, the film thickness is considered to be two parts $H = h + h_s$, where h denotes the smooth part of the film thickness and h_s denotes roughness part of film thickness

3. MODIFIED STOCHASTIC REYNOLDS EQUATION

The two types of roughness pattern are identified. They are namely (a) the one-dimensional longitudinal roughness where roughness pattern are in the form of long narrow ridges and valleys running in the *x* -direction, and (b) the one dimensional transverse roughness where, the roughness striations are in the form of long narrow ridges and valleys running in the z-direction. In the present study, the analysis is restricted to only one- dimensional

ARPN Journal of Engineering and Applied Sciences

longitudinal roughness, because, these two roughness patterns can be made similar, only by rotation of co-ordinate axes.

The film thickness is combined of two parts $H = h(t) + h_s(z,\zeta)$

Considering the expectation on the both sides of the equation (13)in accordance with the Christensen Stochastic approach of the rough surface. The pressure gradient in the direction of roughness and flux perpendicular to it are stochastic variables with zero or negligible variance. The variables with zero variance are the pressure gradient in *x* and the velocity along *y*.

Reynolds's equation for roughness takes the form.

$$\frac{\partial}{\partial x}\left[\frac{dE(p)}{dx}E\left\{f(N,L,H)\right\}\right]+\frac{\partial}{\partial z}\left[\frac{dE(p)}{dz}E\left\{\frac{1}{f(N,L,H)}\right\}\right]=12\mu\frac{dh}{dt} \quad (14)$$

where $E\left\{f\left(N,L,H\right)\right\} = \frac{35}{32C^7}\int_{-c}^{c} f(N,L,H)(c^2-h_s{}^2)^3\,dh_s$

$$E\left\{\frac{1}{f\left(N,L,H\right)}\right\} = \frac{35}{32C^7}\int_{-c}^{c}\frac{(c^2-h_s{}^2)^3}{f(N,L,H)}dh_s$$

The modified Reynolds equation is

$$\frac{\partial^2 E(p)}{\partial x^2} + R\frac{\partial^2 E(p)}{\partial z^2} = \frac{12\mu dh/dt}{E\left\{f(N,L,H)\right\}} \quad (15)$$

where $R = \frac{1}{R_1 R_2}$, $R_1 = E\left\{f\left(N,L,H\right)\right\}$,

$$R_2 = E\left\{\frac{1}{(f(N,L,H)}\right\}$$

Relevant pressure boundary conditions are

$$p(x_1, z_1)=0 \quad (16)$$

where $\frac{x_1^2}{a^2} + \frac{z_1^2}{b^2} = 1$ (17)

Solving (15) by using the boundary condition (16) and (17),the expression for pressure is obtained as

$$E(p) = -\frac{12\mu dh/dt}{E\left\{f(N,L,H)\right\}}\left\{\frac{a^2b^2}{2(Ra^2+b^2)}\left(1-\frac{x^2}{a^2}-\frac{z^2}{b^2}\right)\right\} \quad (18)$$

Introducing non-dimensional quantities as

$$h^* = \frac{h}{h_0}, L^* = \frac{L}{h_0}, a^* = \frac{a}{b}, x^* = \frac{x}{a}, z^* = \frac{z}{b}$$

The non-dimensional film pressure is given by

$$p^* = -\frac{E(p)h_0^3}{\mu(dh/dt)ab} = \left(\frac{a^*}{R^*a^{*2}+1}\right)\frac{6\left(1-x^{*2}-z^{*2}\right)}{E\left\{F(N,L^*,H^*)\right\}} \quad (19)$$

Where

$R^* = \frac{1}{R_1^* R_2^*}$, $R_1^* = E\left\{F\left(N,L^*,H^*\right)\right\}$,

$$R_2^* = E\left\{\frac{1}{F(N,L^*,H^*)}\right\}$$

$$E\left\{F\left(N,L^*,H^*\right)\right\} = \frac{35}{32C^7}\int_{-c}^{c} F(N,L^*,H^*)(C^2-h_s^{*2})^3\,dh_s^*$$

$$E\left\{\frac{1}{F\left(N,L^*,H^*\right)}\right\} = \frac{35}{32C^7}\int_{-C}^{C}\frac{(C^2-h_s^{*2})^3}{F(N,L^*,H^*)}dh_s^*$$

$$F(N,L^*,H^*) = H^{*3} + 12L^{*2}H^* - 6NL^*H^{*2}\coth\left(\frac{NH^*}{2L^*}\right)$$

On integrating the pressure field we get load carrying capacity of the squeeze film

$$W = \int_{-a}^{a}\int_{-\frac{b}{a}\sqrt{a^2-x_1^2}}^{\frac{b}{a}\sqrt{a^2-x_1^2}} E\left(p\right)dzdx \quad (20)$$

The non-dimensional load carrying capacity is given by

$$W^* = -\frac{E(W)h_0^3}{\mu(dh/dt)a^2b^2} = \left(\frac{a^*}{Ra^{*2}+1}\right)\frac{3}{E\left\{F(N,L^*,H^*)\right\}} \quad (21)$$

The expressions for non-dimensional squeeze film time is

$$T^* = -\int_{1}^{t_1/t_0}\frac{E(W)h_0^2\,dt}{\mu a^2b^2} = \int_{1}^{h^*}\left(\frac{3a^*}{Ra^{*2}+1}\right)\frac{dh^*}{E\left\{F(N,L^*,H^*)\right\}} \quad (22)$$

4. RESULTS AND DISCUSSIONS

An investigation has found on the impact of squeeze film between rough elliptical plates lubricated with micro-polar fluid. Three non-dimensional physical parameters have considered for the proposed work such as the couple stress parameter L^*, coupling number N $\left\{=\chi/(\chi+2\mu)\right\}^{1/2}$ and roughness parameter '*C*'. Here '*N*' represents coupling number and it is the coupling between the rotational and Newtonian viscosities. As $\chi \to 0$, $N \to 0$ the characteristics of the bearing expression

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ARPN Journal of Engineering and Applied Sciences

decreases to their analogues in classical Newtonian case. The above mentioned parameters $L^* \left(= L/h_0\right)^{1/2}$ with $L = \left(\gamma/4\mu\right)^{1/2}$ specifies the performance between the configuration of the fluid and bearing.

4.1 Non-dimensional pressure

Figure-2 depicts the variation in non-dimension pressure P^* with x^* for different values of C and L^* when h^*= 0.6, N = 0.2 and a^*= 1.5. It is noted that increase in squeeze film pressure P^* with increasing values of C and L^*. The variation in dimensionless pressure P^* with x^* for different values of N and a^*, when C = 0.3, h^*= 0.6 and L^*= 0.4 is seen in Figure-3. It shows that pressure P^* remarkably enhances with increasing values of a^* and N. The effect is more in rough plates than smooth plates ($C=0$).

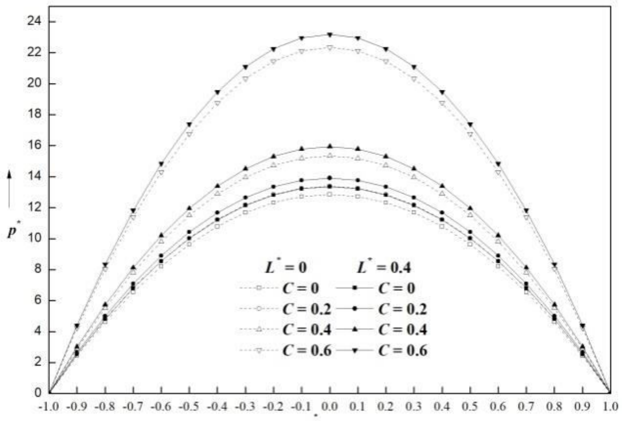

Figure 2: Variation of nondimensional pressure p^* with x^* for different values of C and L^* with $h^*=0.6$, $N=0.2$, $a^*=1.5$ and $z^*=0$.

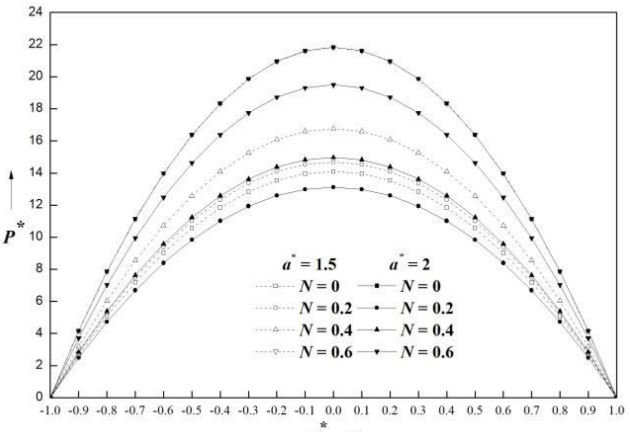

Figure 3: Variation of nondimensional pressure p^* with x^* for different values of a^* and N with $h^*=0.6$, $L^*=0.4$, $C=0.3$ and $z^*=0$.

4.2 Non-dimensional load carrying capacity

From Figure-4 it is understood that the variation of dimensionless W^* with h^* for different values of L^* and C with N = 0.2 and a^* = 1.5. It has been observed that with increasing L^* and C, there is a significant increase in W^*.The dimensionless W^* with h^* for different values of N and a^*, when C = 0.3 and L^*= 0.4 is represented in Figure-5. It is observed that W^* decreasing for increasing values N and a^*. From the proposed work it has been observed that increasing values of N and a^* enhances the load carrying capacity for rough elliptical plates when compared to smooth plates ($C=0$).

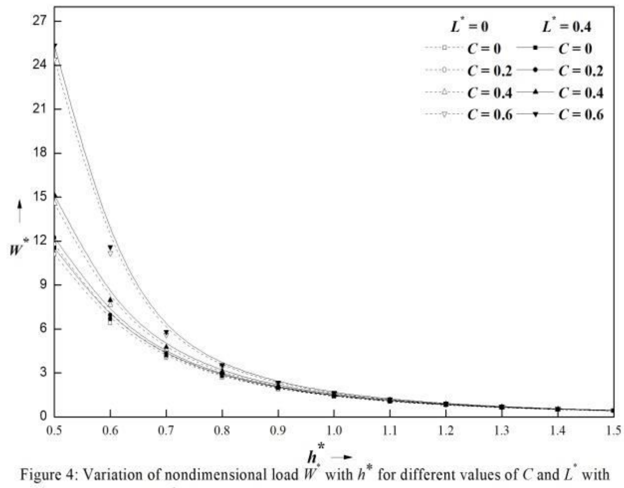

Figure 4: Variation of nondimensional load W^* with h^* for different values of C and L^* with $N=0.2$ and $a^*=1.5$.

4.3 Squeeze film time

When N=0.2 and a^* = 1.5 the variation of dimensionless T^* with h_1^* for different values of L^* and $C$is shown in Figure-6.As the values of L^* and C increases there is a significant increase in squeeze film time T^*. The impact is highly prominent in elliptical rough plates than

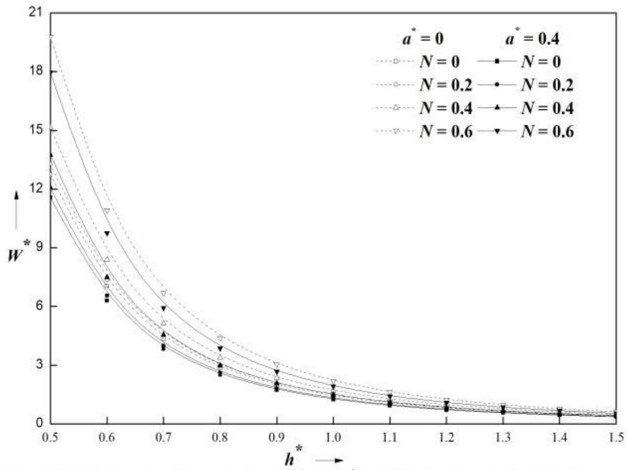

Figure 5: Variation of nondimensional load W^* with h^* for different values of a^* and N with $L^*=0.4$ and $C=0.2$.

Smooth plates. The variation of dimensionless T^* as function of$h_1^*$is shown in Figure-7, for various values of N and a*with C = 0.3 and L^*=0.4.It is observed that T^* increases with larger values of N and a*.

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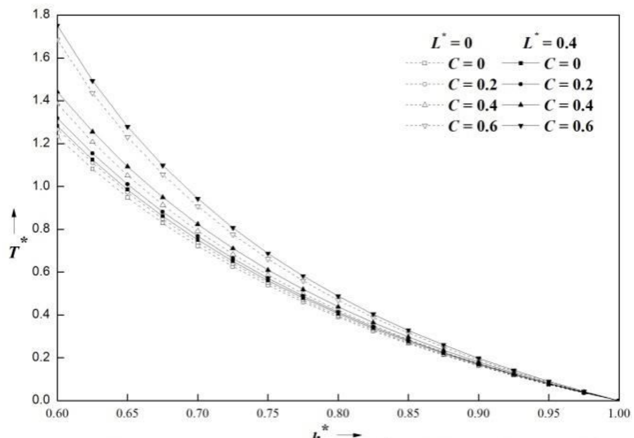

Figure 6: Variation of nondimensional time T^* with h^* for different values of C and L^* with $N = 0.2$ and $a^* = 1.5$.

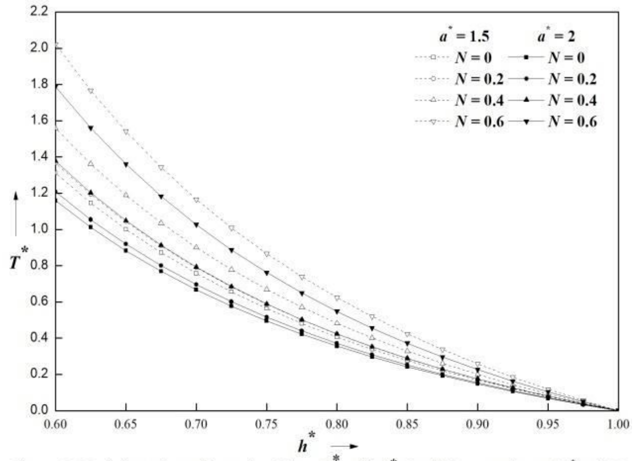

Figure 7: Variation of nondimensional time T^* with h^* for different values of a^* and N with $L^* = 0.4$ and $C = 0.3$.

5. CONCLUSIONS

The effect of surface roughness on the squeeze film characteristics of rough elliptical plates is presented on the basis of Eringen's micro polar fluid theory and Christensen Stochastic theory for the study of rough surfaces, the modified form of Stochastic Reynold's equation is derived. As the micro polar fluid parameter $N \rightarrow 0$ and $L^* \rightarrow 0$ the characteristics of the squeeze film reduces to Newtonian case and as $C \rightarrow 0$ these characteristics reduces to the smooth case. On the basis of results obtained, the following conclusions were drawn.

- It is observed that when roughness parameter C increases, squeeze film time, pressure and load carrying capacity increases.
- The non dimensional couple stress parameter L^*leads to increase in squeeze film time,pressure and load carrying capacity.

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