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MHD NATURAL CONVECTION FLOW PAST A MOVING VERTICAL PLATE WITH RAMPED TEMPERATURE

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ABSTRACT

An investigation has been carried out numerically to study MHD natural convection flow of an incompressible, electrically conducting, radiative, viscous dissipation and heat absorbing fluid past a moving vertical plate with ramped temperature. Solutions of the governing partial differential equations for primary velocity, secondary velocity and temperature are obtained by using Finite element method. The influence of Magnetic parameter M^2, thermal Grashof number Gr, Hall current parameter m, viscous dissipation Ec and thermal radiation parameter Nr on primary velocity, secondary velocity and temperature are computed numerically and represented graphically. Effect of pertinent flow parameters on shear stress and rate of heat transfer are displayed in a tabular form. Various comparisons with previously published work are performed and the results are found to be in excellent agreement.

Keywords: MHD, natural convection, viscous dissipation, ramped temperature, FEM.

INTRODUCTION

Magneto hydrodynamics is concerned with the study of mutual interaction of magnetic fields and electrically conducting fluids in motion and the fluids must be non-magnetic which limits us to liquid metals such as mercury, gallium, sodium, molted-iron and hot ionized gases(plasma), strong electrolytes etc. The magnetic field influence many natural and man-made flow. There is the terrestrial magnetic field which is maintained by fluid motion in earth's core, the solar magnetic field which generates sunspots and solar flares and the galactic magnetic field which is thought to influence the formation of stars from interstellar clouds. For the last several decades this subject has been interested by many scientists and engineers due to its fascination and importance in various technology devices and for understanding the diverse cosmic phenomena. There are numerous examples of applications of MHD principles like MHD generators, MHD pumps, MHD flow meters stir and levitate liquid metals etc. Convection problems of electrically conducting fluid in presence of transverse magnetic field have got much importance because of its wide applications in Geophysics, Astrophysics, Plasma Physics, Missile technology etc. MHD principles find its applications in Medicine and Biology also. The present stage of MHD is due to the pioneer contributions of several notable authors like Cowling [1] has studied Magneto hydrodynamics. Raptis and Kafousias [2] analyzed the effects of magnetic field on steady free convection flow in a porous medium bounded by an infinite vertical plate. Chamkha [3] considered unsteady hydromagnetic two-dimensional convective heat and mass transfer flow past a semi infinite vertical permeable moving plate with temperature dependent heat absorption.

Natural convection flow induced due to thermal buoyancy force in fluid saturated porous medium has gained considerable attention of several researchers in the past due to its frequent occurrence in nature and due to its varied and wide industrial applications viz. drying of porous solids, thermal insulators, geothermal reservoirs, heat exchanger devices, enhanced recovery of oil and gases, underground energy transport etc. Keeping in view the importance of such fluid flow Cheng and Minkowycz [4] obtained similarity solution for natural convection flow from a vertical plate embedded in a fluid saturated porous medium. Barik *et al.* [5] have examined unsteady free convective MHD flow and mass transfer through porous medium in a rotating system with fluctuating heat source/sink and chemical reaction.

Viscous dissipation occurs in natural convection in various devices. Such dissipation effects may also be present in stronger gravitational fields and in process wherein the scale of the process is very large, e.g., on larger planets, in large masses of gas in space, and in geological processes in fluids internal to various bodies. Gebhart [6] studied the importance of viscous dissipative heat in free convective flows on a vertical surface subject to isothermal and uniform flux surface conditions. Ganesan and Palani [7] studied transient unsteady viscous dissipative fluid flow past a semi-infinite isothermal inclined plate. Sivaiah and Srinivasaraju [8] have discussed Finite element solution of heat and mass transfer flow with Hall current, heat source, and viscous dissipation.

In all these investigations, the analytical or numerical solutions are obtained by considering conditions for the velocity and temperature at the plate as continuous and well defined. However, there exist several problems of practical interest which may require non-uniform or arbitrary conditions at the plate. Keeping in view of this fact, several researchers investigated free convection flow from a vertical plate with step discontinuities in the surface temperature. Mention may be made of research studies. Seth *et al.* [9] investigated the effects of thermal radiation and rotation on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. Seth *et al.* [10] have studied the Effects of Hall current, radiation and rotation on natural convection heat and mass transfer flow past a moving vertical plate. Seth *et al*. [11] have analyzed Unsteady MHD free convection flow with Hall effect of a radiating and heat absorbing fluid past a moving vertical plate with variable ramped temperature Seth[12] have examined MHD natural convection flow with hall effects, radiation and heat absorption over an exponentially accelerated vertical plate with ramped temperature. Siva Reddy *et al*. [13] have found transient approach to heat absorption and radiative heat transfer past an impulsively moving plate with ramped temperature.

FORMULATION OF THE PROBLEM

Consider unsteady hydro magnetic free convection flow of a viscous, incompressible, electrically conducting, optically thick radiating and heat absorbing fluid past a moving infinite vertical flat plate with variable ramped temperature. We choose the Cartesian coordinate system (x', y', z') in such a way that x'-axis is along the vertical plate in upward direction, y'-axis is normal to the plane of the plate directed into the fluid region and z'-axis is normal to $x'y'$-plane. A uniform transverse magnetic field of strength B_0 is applied in a direction parallel to y'-axis. Initially, i.e. at time $t \le 0$, both the plate and surrounding fluid are at rest and maintained at uniform temperature T'_∞. At time $t > 0$, the plate starts moving along x' direction with a velocity $U(t') = a't'$ (a' being arbitrary constant) and at the same time temperature of the plate is raised to $T'_\infty + (T'_w - T'_\infty)(t'/t_0)$ when $0 < t' \le t_0$ and it is maintained at uniform temperature T'_w when $t' > t_0$ (t_0 being critical time for rampedness). Physical model of the problem is shown in Figure-1.

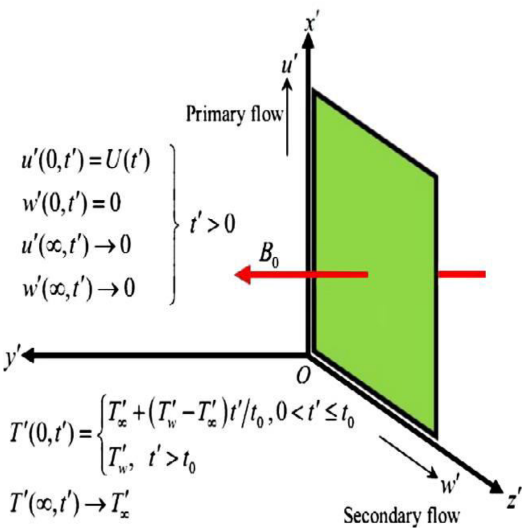

Figure-1. Physical model of the problem.

Since plate is of infinite extent along x' and z' directions, all physical quantities except pressure depend on y' and t' only. Induced magnetic field produced by the fluid motion is neglected in comparison to applied one. This is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in various industrial processes (Cramer and Pai [14]).Since no external electric field is applied into the flow field so the effect of polarization of fluid is negligible which corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai [14]).

With the assumptions made above, the governing equations for the fluid flow problem taking Hall current into account, under Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(u' + mw') + g\beta\left(T' - T'_\infty\right) \quad (1)$$

$$\frac{\partial w'}{\partial t'} = \upsilon \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(mu' - w') \quad (2)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p}\left(T' - T'_\infty\right) - \frac{1}{\rho c_p}\frac{\partial q'_r}{\partial y'} + \frac{v}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \quad (3)$$

Initial and boundary conditions for the problem are specified as

$$t' \le 0:\ u' = 0,\ w' = 0,\ T' = T'_\infty, \quad for all \quad y' \ge 0$$
(4)

$$t' > 0: u' = a't',\ w' = 0$$
$$T' = \begin{cases} T'_\infty + \left(T'_w - T'_\infty\right)t'/t_0 & at\ \ y' = 0\ when\ \ 0 < t' \le t_0, \\ T'_w, & at\ \ y' = 0\ when\ t' > t_0 \end{cases} \quad (5)$$

$$t' > 0: u' \to 0, u' \to 0 T' \to T'_\infty, \quad as \quad y' \to \infty \quad (6)$$

For an optically thick gray fluid, the radiative heat flux q'_r is approximated by Rosseland approximation (Howell *et al.*) which is given by

$$q'_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \quad (7)$$

It is assumed that the temperature difference between fluid in the boundary layer region and free stream is very small so that T'^4 is may be expanded as a linear function of T'. Expanding T'^4 in Taylor series about T'_∞ and neglecting higher order terms

$$T'^4 \approx 4T_\infty'^3 T' - 3T_\infty'^4 \tag{8}$$

Making use of Equations (7) and (8) in Equation (3), we Obtain

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\left(1+\frac{16\sigma^* T_\infty'^3}{3kk^*}\right)\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p}(T'-T_\infty') + \frac{v}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{9}$$

We introduce following non dimensional quantities and flow parameters to present Equations (1), (2) and (9) along with initial and boundary conditions (4)-(6) in non-dimensional form

$$\left.\begin{array}{l} \eta = \frac{U_0 y'}{\upsilon},\ t = \frac{U_0 t'}{\upsilon}, u = \frac{u'}{U_0}, w = \frac{w'}{U_0}, T = \frac{(T'-T_\infty')}{(T_w'-T_\infty')}, Gr = \frac{g\beta v(T_w'-T_\infty')}{U_0^3}, \\ M^2 = \frac{\sigma B_0^2}{\rho U_0^2},\ Nr = \frac{16\sigma^* T_\infty'^3}{3kk^*}, \Pr = \frac{v\rho c_p}{k},\ \varphi = \frac{vQ_0}{\rho c_p U_0^2}, Ec = \frac{U_0^2}{c_p (T_w'-T_\infty')} \end{array}\right\} \tag{10}$$

Making use of Equation (10), Equations (1), (2) and (9) in non-dimensional form, reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \eta^2} - \left(\frac{M^2}{1+m^2}\right)(u+mw) + GrT \tag{11}$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial \eta^2} + \left(\frac{M^2}{1+m^2}\right)(mu-w) \tag{12}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}(1+Nr)\frac{\partial^2 T}{\partial \eta^2} - \varphi T + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{13}$$

Where $u', w', v, \sigma, \rho, g, \beta, T', m, k, c_p and\ Q_0$ are fluid velocity in x'-direction, fluid velocity in z'-direction, kinematic coefficient of viscosity, electrical conductivity, density, acceleration due to gravity, coefficient of thermal expansion, fluid temperature, Hall current parameter, conductivity, specific heat at constant pressure and heat absorption coefficient respectively.

Here $M^2, m, Gr, \Pr, Nr, Q, Ec, and\ t$ are magnetic parameter, Hall current parameter, thermal Grashof number, Prandtl number, thermal Radiation parameter, dimensionless heat absorption coefficient, Eckert number and time respectively.

Initial and boundary conditions in non-dimensional form, are given by

$$\left.\begin{array}{l} u = w = 0,\ T = 0, \quad for\ \ y \ge 0\ \ and\ t \le 0, \\ u = 1, w = 0\ \ at\ \ y = 0\ \ for\ t > 0, \\ T = t\ \ at\ y = 0\ \ for\ 0 < t \le 1, \\ T = 1\ \ at\ \ y = 0\ \ for\ t > 1, \\ u \to 0,\ w \to 0, T \to 0\ \ \ as\ y \to \infty\ \ \ for\ t > 0 \end{array}\right\}$$

(14)

METHOD OF SOLUTION

Finite element method has been used to solve equations (11) to (13) subject to initial and boundary conditions (14).

The steps involved in the finite-element analysis are as follows:

a) Division of the domain into linear elements, called the finite element mesh.
b) Generation of the element equations using variational formulations.
c) Assembly of the element equations as obtained in step (ii).
d) Introduction of the boundary conditions to the equations obtained in step (iii).
e) Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gaussian elimination, LU Decomposition method etc. The details of the method used here can be studied in the paper given by Bathe [15] and Reddy [16].

VALIDATION OF RESULTS

In order to verify the accuracy of the present method, the results are compared with those cases reported by Seth *et al.* [11] in the absence of Eckert number (i.e. $Ec = 0$). As shown in Table 1 the comparisons are found to be in a very good agreement. Therefore, the developed code can be used with great confidence to study the problem considered in this paper.

Table-1. Comparison of $-\tau_x$, τ_z and Nu when $(Ec = 0)$.

Results by Seth *et al.* [11]				Present results		
Nr	$-\tau_x$	τ_z	Nu	$-\tau_x$	τ_z	Nu
2	3.1373	1.1170	0.4586	3.13729	1.1169	0.45859
4	3.0932	1.1338	0.3553	3.09319	1.13379	0.35530
6	3.0685	1.1436	0.3002	3.06849	1.14360	0.30019

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RESULTS AND DISCUSSIONS

With the help of FEM, the nonlinear partial differential equations (11)–(13) together with the initial and boundary conditions (14) are numerically solved. In order to study the behavior of primary velocity, secondary velocity and temperature functions, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics and the results are reported in Figures 2-19.

The influence of Magnetic parameter M^2 on primary velocity u and secondary velocity w is displayed in Figures 2 and 3 for both ramped temperature and isothermal plates. Fluid velocities u and w decrease on increasing M^2. Physically, it is true due to the fact that the primary and secondary fluid motions are retarded due to application of transverse magnetic field. This phenomenon clearly agrees to the fact that Lorentz force that appears due to interaction of the magnetic field and fluid velocity resists the fluid motion.

The effect of thermal Grashof number Gr on primary velocity u and secondary velocity w is depicted in Figures 4 and 5 for both ramped temperature and isothermal plates. Grashof number is a dimensionless number which approximates the ratio of the buoyancy to viscous force acting on a fluid. From these Figures it is understood that Fluid velocities u and w increase on increasing Gr. Thermal Grashof number signifies the relative strength of thermal buoyancy force to viscous force. Gr increases when thermal buoyancy force increases. This implies that thermal buoyancy force tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

The impact of Hall current parameter m on primary velocity u and secondary velocity w is portrayed in Figures 6 and 7 for both ramped temperature and isothermal plates. Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. From Figures 6 and 7 it is clear that Fluid velocities u and w increase on increasing m .This is due to the fact that hall current induces secondary flow which accelerates fluid velocity.

The effect of viscous dissipation Ec on primary velocity u, secondary velocity w and temperature T is exhibited in Figures 8, 9 and 10 for both ramped temperature and isothermal plates. Viscous dissipation is the irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat. From Figures 8 and 9 it is clear that Primary velocity u and secondary velocity w increase on increasing Ec .This happens due to the reason that greater viscous dissipative heat causes a rise in the velocity. From Figure 10 it is observed that fluid temperature increases on increasing Ec .

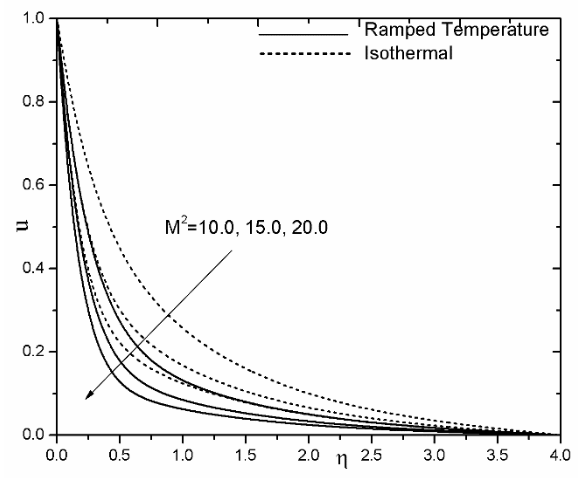

Figure-2. Primary velocity profile for M^2.

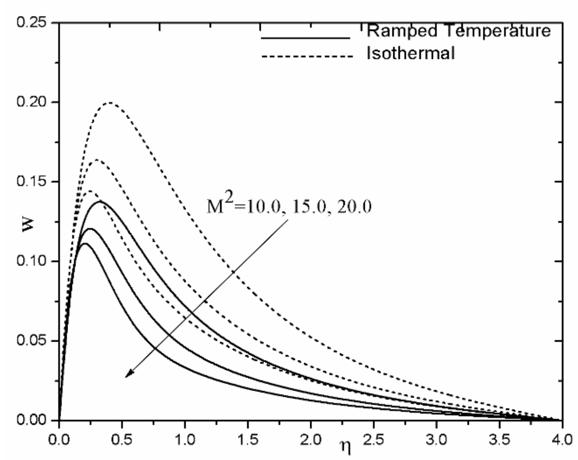

Figure-3. Secondary velocity profile for M^2.

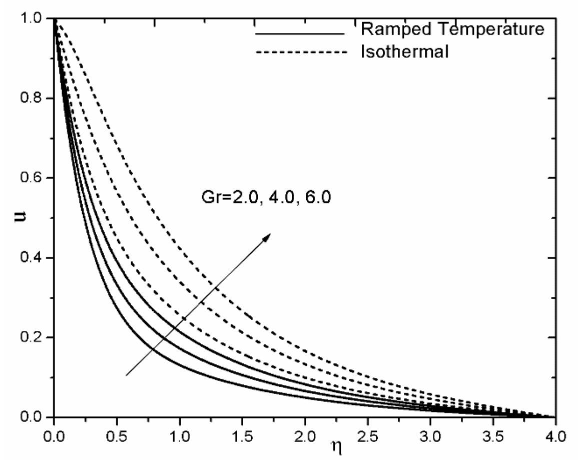

Figure-4. Primary velocity profile for Gr.

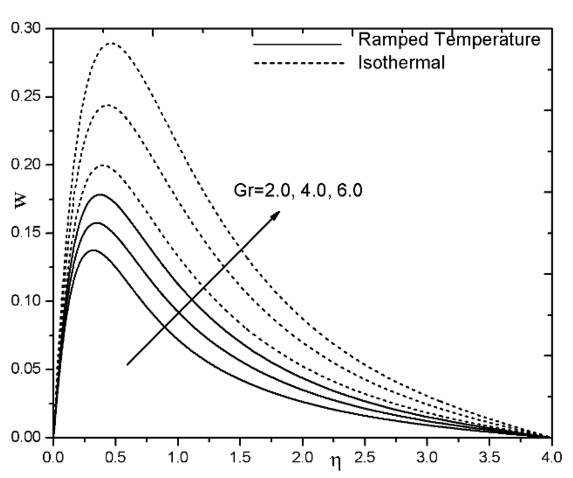

Figure-5. Secondary velocity profile for Gr.

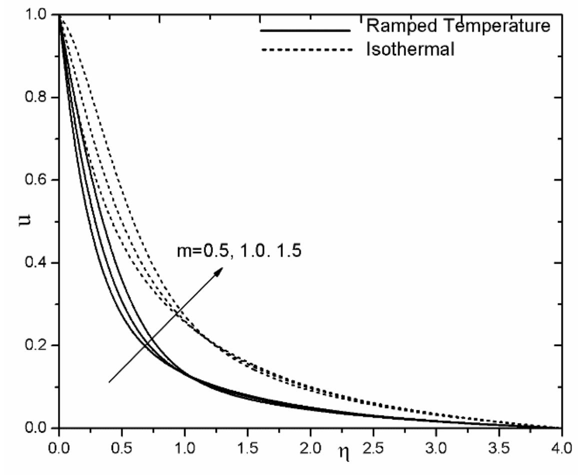

Figure-6. Primary velocity profile for m.

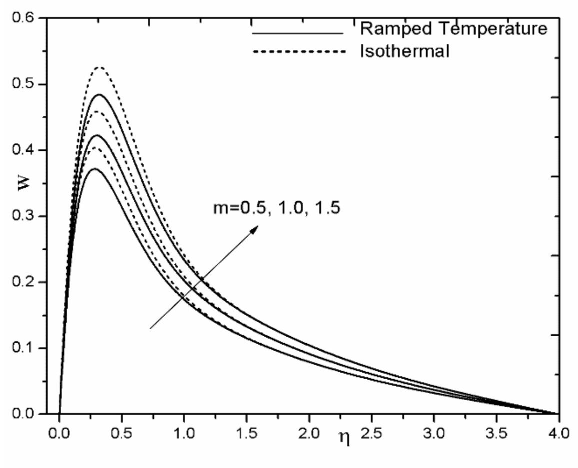

Figure-7. Secondary velocity profile for m.

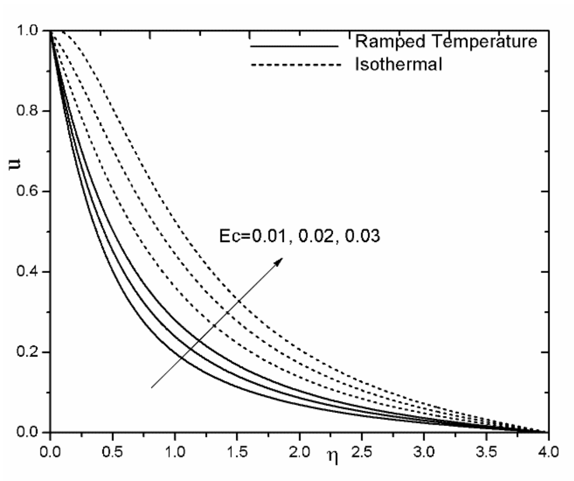

Figure-8. Primary velocity profile for Ec.

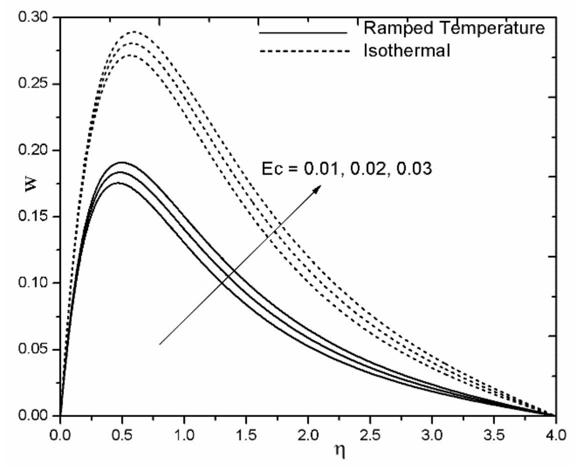

Figure-9. Secondary velocity profile for Ec.

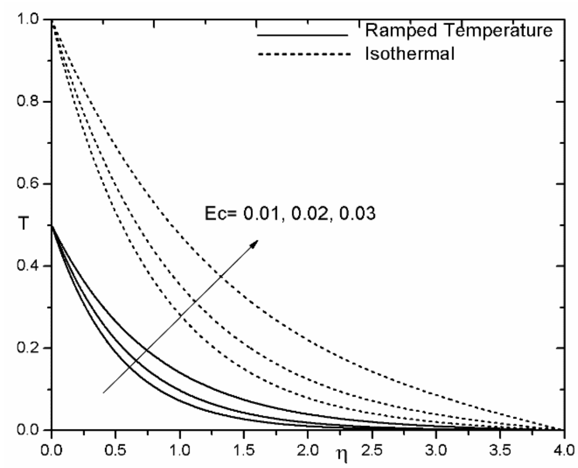

Figure-10. Temperature profile for Ec.

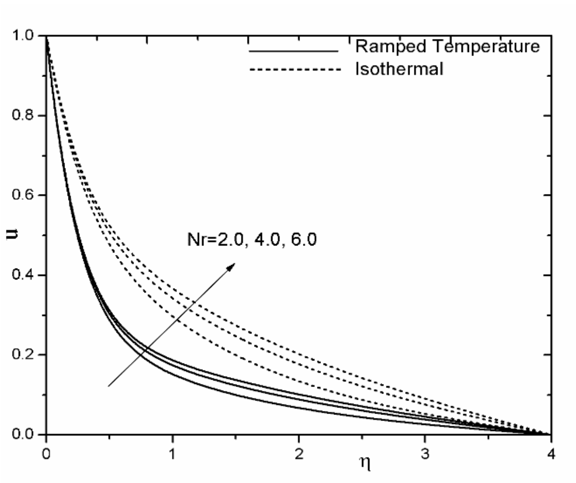

Figure-11. Primary velocity profile for Nr .

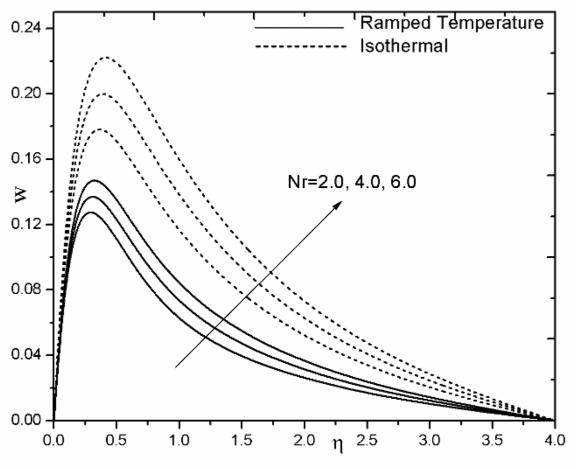

Figure-12. Secondary velocity profile for Nr .

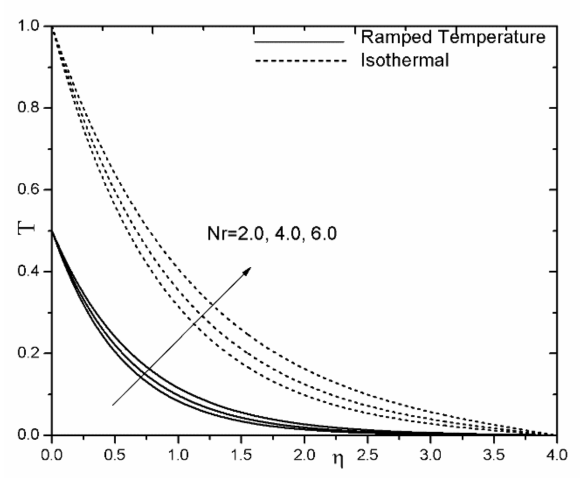

Figure-13. Temperature profile for Nr .

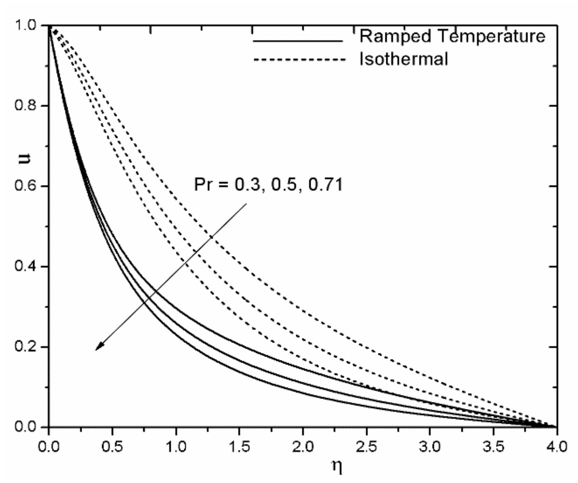

Figure-14. Primary velocity profile for Pr .

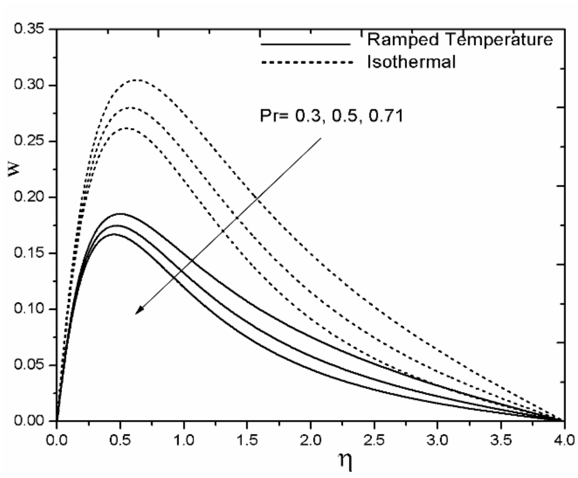

Figure-15. Secondary velocity profile for $\Pr$.

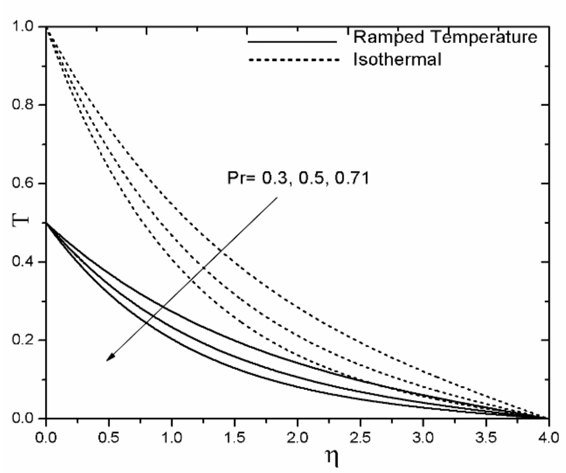

Figure-16. Temperature profile for Pr .

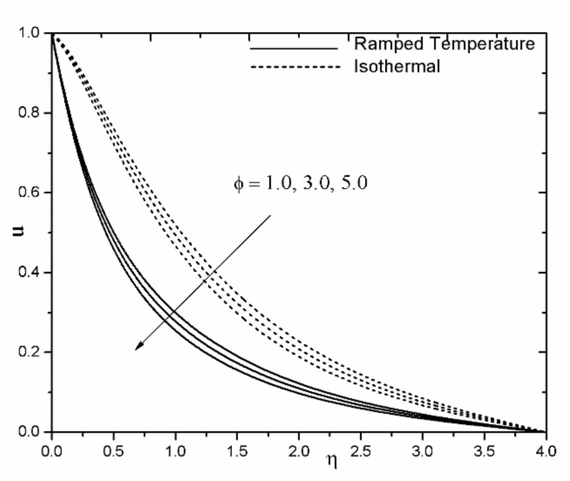

Figure-17. Primary velocity profile for ϕ.

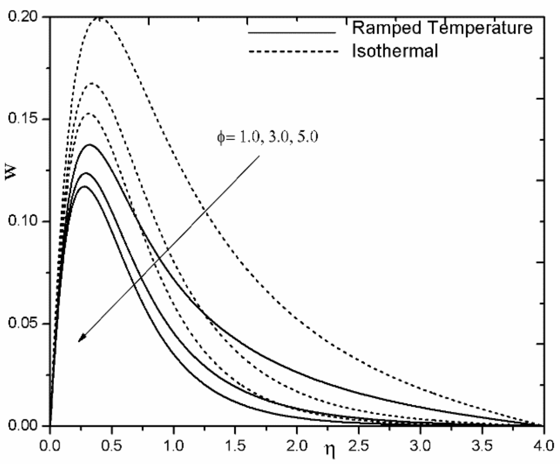

Figure-18. Secondary velocity profile for ϕ.

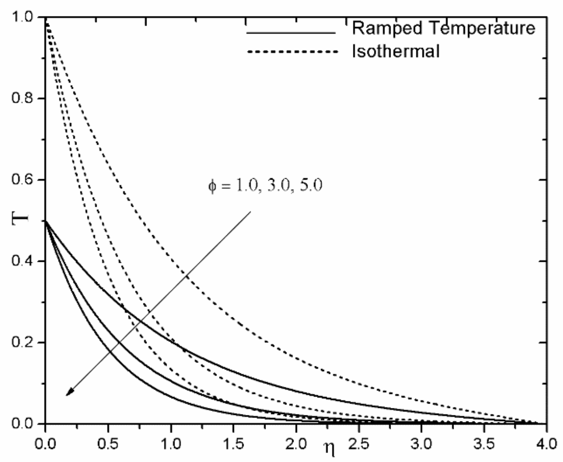

Figure-19. Temperature profile for ϕ.

The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature.

The effect of thermal radiation Nr on primary velocity u, secondary velocity w and temperature T is exhibited in Figures 11, 12 and 13 for both ramped temperature and isothermal plates. Thermal radiation is electromagnetic radiation generated by the thermal motion of charged particles in matter. From these Figures it is conveyed that primary velocity u, secondary velocity w and temperature T increase on increasing Nr. Nr defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in increasing velocity and temperature within the boundary layer.

Figures 14-16 express the influence of the Prandtl number $\Pr$ on velocity and temperature profiles for both ramped temperature and isothermal plates. The Prandtl number $\Pr$ is a ratio of momentum diffusivity to the thermal diffusivity. Figures 14 and 15 portray the decrease in primary and secondary velocities on increasing the Prandtl number. It is due to the reason that Prandtl number causes an increase in the viscosity of the fluid which makes the fluid thick and it leads to a decrease in the velocities of the fluid for both ramped temperature and isothermal plates. Figure 16 depicts that the fluid temperature diminishes on increasing the Prandtl number. The reason is that smaller values of Pr are equivalent to enlargement in the thermal conductivity of the fluid and then heat is able to diffuse away from the heated surface more quickly for higher values of Pr. Hence, in the case of a lesser Prandtl number, the thermal boundary layer is substantial and the rate of heat transfer is reduced.

Figures 17-19 present the influence of the heat absorption parameter ϕ on the velocities and temperature profiles for both ramped temperature and isothermal plates. It is evident from these Figures that heat absorption tends to retard the velocities and temperature profiles. Actually speaking, the heat absorption (thermal sink) effect has the tendency to reduce the fluid temperature. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity. These behaviors are clearly obvious from Figures 17-19 in which the velocity and temperature distributions decrease as ϕ increases. It is also observed that both the hydrodynamic (velocity) and the thermal (temperature) boundary layers decrease as the heat absorption effects increase.

CONCLUSIONS

Present investigation deals with MHD natural convection flow past a moving vertical plate with ramped temperature. The significant results for both ramped temperature and isothermal plates are summarized below:

- Primary and secondary velocities increase on increasing thermal Grashof number Gr, Hall current parameter m, viscous dissipation Ec and thermal radiation parameter Nr and they have reverse tendency on increasing Magnetic parameter M^2, Prandtl number Pr and heat absorption ϕ.

- Temperature increases on increasing viscous dissipation Ec and thermal radiation parameter Nr and temperature profile experiences a decline with the increasing values of Prandtl number Pr and heat absorption ϕ.

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