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ACHIEVING SENSITIVITY ROBUSTNESS IN THE OPTIMUM DESIGN OF A CONICAL VESSEL - AN ILLUSTRATION

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ABSTRACT

Optimum Design generally becomes rigid or brittle; in a sense that no change or variation in the set value of design variables and parameters is tolerated. A change or variation in set value of design variables and/or design parameters may cause improper functioning or even failure in some critical cases. The Optimum Design is generally so sensitive to change or variability of design variables that it does not tolerate any change or variation though the change is very small and even of the level of geometric tolerances. The main reason behind this is the variation in design variables and parameters get transmitted to the design function causing variation in it. Variation of design function may result in improper functioning or even failure. In this context Robust Optimum Design is that Optimum Design which tolerates variations. The key concept to achieve robustness is to minimize the effect of transmitted variation (often called as 'Induced Variation') to such an extent that it is hardly noticeable. If the induced variation is minimized to such an extent then despite variations no improper functioning and no failure is ensured up to a certain extent. Thus the design becomes robust and therefore it is called as a 'Sensitivity Robust Optimum Design'. In this paper this concept is illustrated with the help of a fictitious problem of designing a conical shaped vessel.

Keywords: induced variation, robust optimum design, sensitivity robustness, transmitted variation.

1. INTRODUCTION

For most of the design problems, an infinite number of possible design solutions can be found which are designated as 'Adequate Designs'. These are adequate in a sense that these satisfy functional requirements while remaining within the confines of existing limitations.

Any mechanical element is associated with it certain inherently unavoidable undesirable effects like stresses, deflections, vibrations, weight, cost etc. and certain desirable effects like power transmission capability, energy absorption capability, usable length of life, etc. As any design problem can have number of adequate design solutions, to find the best out of them which will result in maximum benefit or in order to obtain a more explicit method of designing, an overall objective of the design should be defined clearly. Depending upon the problem in hand, an objective of the design in form of either to minimize the most significant undesirable effect or to maximize the most significant desirable effect can be defined. Once an objective of the design is clearly defined, it results in an explicit design procedure to arrive at a solution which is the best possible, most suitable and most beneficial. Such a design is called as an 'Optimum Design' [6] & [7].

The Optimum Design Solution is in form of a set of values each one for each individual design variable. An 'Optimum Design' becomes rigid or brittle in a sense that it tolerates no change or variation in the set value of design variables or design parameters. A change or variation in set values of design variables and/or design parameters may cause improper functioning or even failure in some critical cases. The Optimum Design is generally so sensitive to the changes or variations that it does not tolerate any change or variation though the change is very small and even of the level of geometric tolerances. In this context 'Robust Optimum Design' is that Optimum Design which tolerates variations. The variations (which also include the geometric tolerances) are the expected deviation of design variables and/or parameters form their set values.

One of the key concepts of robust optimum design is variations in variables and parameters get transmitted to the design function causing variations in it. The variation of the design function, which is due to variation of design variables and/or parameters, is called as 'Induced Variation' or 'Transmitted Variation' [1], [2] & [3].

If any how the effect of this Transmitted Variation or Induced Variation in the design function is taken care of such that despite variations no improper functioning and no failure is ensured up to a certain extent then the resultant design can be assumed to be feasible despite variations and tolerant to the variations and hence a 'Feasibility Robust Optimum Design'. If the Transmitted Variation or the Induced Variation is reduced to minimum possible level and hence variation of design function became hardly noticeable. Then it can be assumed that the variations in variables and parameter have become tolerable by the design up to a certain extent and the resultant design is a robust design. The act of minimizing the Induced Variation is nothing but to reduce the sensitivity of the design to the variability. The design with minimized sensitivity is referred as a 'Sensitivity Robust Optimum Design' [4] & [5].

In this paper the concept of 'Sensitivity Robust Optimum Design' is illustrated with the help of a fictitious problem of designing a conical shaped vessel.

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2. FORMULATION OF SENSITIVITY ROBUST OPTIMUM DESIGN

A. Problem definition

A metallic vessel is required to be designed for storing a peculiar liquid. The vessel is to be used in an important scientific experiment. The vessel should have storage capacity approximately equal to but not less than 2520 cm^3. The shape of the vessel is decided to be of a cone. It is required to have the surface area of the vessel (excluding base area of the cone) not less than 2800 cm^2. Base radius of the vessel should not be less than 5cm and it should not exceed a limit of 15 cm. The height of the vessel should not exceed a maximum value of 100 cm and it should not be less than 20 cm. A factor called as 'Factor K', whose value depends upon Radius 'R' and Height 'H' of the vessel should be maintained at minimum possible level. The expected tolerance, both on the base radius as well as the height of the vessel is $\pm$ 0.1 cm. Variability or the tolerance on the dimensions 'R' and 'H' will vary the calculated value of 'Factor K' and it is equally rather bit more important to minimize fluctuation in the value of 'Factor K' as some further critical calculations involve the calculated value of 'Factor K'. So it is expected to design the vessel (i.e. to decide values of radius 'R' and height 'H' of the vessel) with an aim of minimizing the 'Factor K' as well as minimizing variations in calculated value of 'Factor K'. The 'Factor K' is calculated by an empirical relation given by,

Factor $K = R^{4.59} + H^{1.75}$

B. Problem solution

For the problem specified, the dimensions of the vessel (i.e. radius R and height H) are to be calculated with an aim of minimizing the Factor K while making sure in the design that the volume of the vessel is at least equal to 2520 cm^3 and surface area of it is not less than 2800 cm^2. It is to make sure in the design that the radius of the vessel is in the range of 5 cm to 15 cm while the height of the vessel is to be maintained in between 20 cm to 100 cm. Up to this point it is a simple optimization problem. In the later part of the problem statement it is been stated that it is equally rather bit more important to reduce or minimize the Induced Variation in the calculated value of Factor K. To minimize Induced Variation means to minimize the sensitivity of the design towards variability of the design variables. So in the later part of the problem statement it is expected to achieve Sensitivity Robustness in the design. Therefore as a whole it is a problem of 'Sensitivity Robust Optimum Design'.

a) Strategy for solving the problem: While solving this problem first of all a mathematical model of the specified problem will be formulated. For that all the conditions specified in the problem statements will be rewritten in form of mathematical equations. A suitable method will be selected based on the nature of the mathematical model to solve the problem. An ordinary 'Optimum Design' solution will be devised by solving the mathematical model. Later on sensitivity of the ordinary optimum design will be checked and if required due modifications will be made in the design so as to achieve a Sensitivity Robust Optimum Design.

b) Mathematical model: Using the conditions as stated in the problem statement first of all a mathematical model of the design problem is formulated as below

The conditions stated in the problem statement are as below:

Minimize Factor K of the conical shaped vessel.

Volume of the vessel 'V' should be approximately equal to but not less than 2520 cm^3.

Surface area 'S' of the vessel (excluding base area) should be more than 2800 cm^2.

Base radius 'R' should be in the range of 5 cm to 15 cm.

Height of the tank 'H' should be in the range of 20 cm to 100 cm.

In short the mathematical model will be

Minimize Factor $K = R^{4.59} + H^{1.75}$

When $V \geq 2520$ cm^3

And $S \geq 2800$ cm^2.

Also 5 cm $\leq R \leq$ 15 cm

And 20 cm $\leq H \leq$ 100 cm

c) Optimum design: While optimizing it is required to decide a value for radius R and a value for height H and so these are the design variables of the problem. As the number of variables in the design is two therefore this optimization problem can be solved using graphical method of optimization.

In the method of graphical optimization it is required to plot the various equations in the mathematical model so as to achieve the feasible design region of the problem. For the problem in hand it is decided to plot the radius R on X-axis and height H on Y-axis.

The Mathematical model comprises six conditions (functional requirements and/or constraints) and each condition signifies a line on the plot. Each condition signifies a boundary limit of the feasible design region.

The first condition defined is $V \geq 2520$ cm^3.

The volume of the cone is given by $V = \frac{1}{3} \pi R^2 H$

Therefore $\frac{1}{3} \pi R^2 H = 2520$ cm^3.

And based on this condition height H is given by, $H = 2406.42 / R^2$.

Value of H for different values of R can be calculated as shown in Table-1.

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Table-1. Value of H against Value of R (**S**o as to Result Volume V ≈ 2520 cm^3).

Value of R (cm)	**Value of H** (cm)
3	267.38
4	150.40
5	96.26
6	66.85
7	49.11
8	37.60
9	29.71
10	24.06
11	19.89
12	16.71
13	14.24
14	12.28
15	10.70
16	9.40

Using the values of R and H from Table-1 a line can be drawn on H against R plot as shown by line L1 in Figure-1.

The second condition is $S \geq 2800$ cm^2.

The surface area of the cone (excluding base area) is given by $S = 2\,\pi\,R\,H_s$

Where H_s is the slant height of the cone given by,

$H_s = (R^2+H^2)^{1/2}$

Therefore $S = 2\,\pi\,R\,(R^2+H^2)^{1/2}$

And based on this condition height H is given by,

$H = [(198589.52/R^2) - R^2]^{1/2}$

Value of H for different values of R can be calculated as shown in Table-2.

Table-2. Value of H against value of R (**S**o as to Result Volume S ≈ 2800 cm^2)

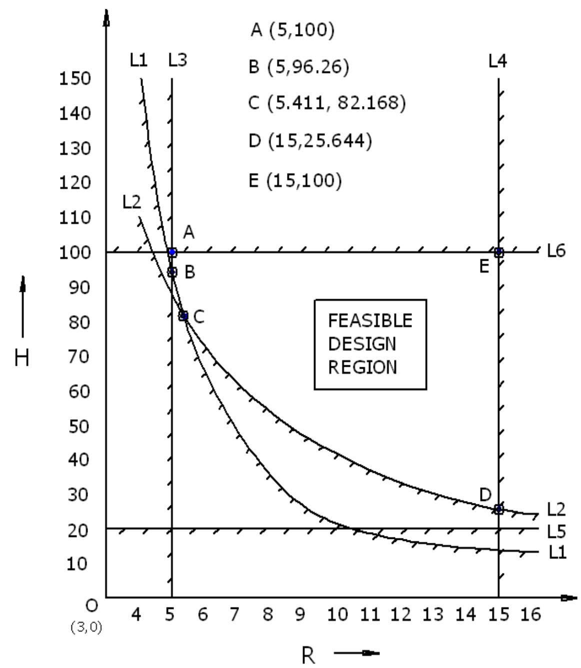
Value of R (cm)	**Value of H** (cm)
4	111.3366
5	88.98641
6	74.02956
7	63.27596
8	55.12677
9	48.69007
10	43.42689
11	38.9902
12	35.1439
13	31.71886
14	28.58692
15	25.6441

Using the values of R and H from Table-2 a line can be drawn on H against R plot as shown by L2 in Figure-1.

Using the condition as stipulated in the mathematical model viz. 5 cm ≤ R (i.e. R ≥ 5 cm) and R ≤ 15 cm two more lines L3 and L4 can be drawn on H against R plot as shown in Figure-1.

Using the condition 20 cm ≤ H ≤ 100 cm, another two lines L5 and L6 fulfilling the conditions H ≥ 20 cm and H ≤ 100 cm respectively can be drawn as shown in Figure-1.

Thus the six lines drawn on the plot indicate boundaries of a confined region called as the feasible design region as shown in Figure-1. Any point inside this confined design region satisfies all the functional requirements as well as constraints of the design and so it signify a feasible design point.



L1: $V = \frac{1}{3}\,\pi\,R^2H = 2520$ cm^3 ⇒ $H = 2406.42 / R^2$

L2: $S = 2\,\pi\,R\,(R^2+H^2)^{1/2} = 2800$ cm^2

⇒ $H = [\,(98589.52/R^2) - R^2\,]^{1/2}$

L3: $R \geq 5$ cm L4: $R \leq 15$ cm

L5: $H \geq 20$ cm L6: $H \leq 100$ cm

Figure-1. Constraint boundaries and feasible design region.

The area enclosed by the six constraint boundaries (i.e. by the lines L1, L2, L3, L4, L5 and L6) is the feasible design region and it is called so because any point inside this region represents a feasible or an adequate design solution (or possible design solution which satisfies all the conditions as stated in problem statement). The points A, B, C, D and E are the points of intersection of the constraint boundaries with each other and these are called as points of extremity. As can be seen from the Figure-1, the feasible design region is convex and

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therefore from the philosophy of graphical optimization, one of the points of extremity represents the optimum design solution for such situation.

Each of the points of extremity is a point of intersection of two constraint boundaries and mathematical equation for each constraint boundary is available. Therefore solving two constraint boundary equations simultaneously the co-ordinates (i.e. values of R and H) for different points of extremity can be calculated as shown in Figure-1.

Using these co-ordinates i.e. values of R and H, the Factor K for the conical storage vessel can be calculated at each of the point of extremity. The objective function of the design is to minimize the Factor K and therefore the point of extremity bearing least value of Factor K will be declared as the Optimum Design solution.

Factor K is given by,
Factor K = $R^{4.59} + H^{1.75}$
Using R = 5 cm and H = 100 cm, i.e. the co-ordinates of point A, Factor K at point A can be calculated as
Factor K = $5^{4.59} + 100^{1.75}$
Factor K = 4777.648801

On the similar lines taking into account the co-ordinates of rest of the points of extremity, the Factor K at the remaining points of extremity can be calculated as shown in Table-2.

Table-3. Values of factor k at different points of extremity.

Point of extremity	Coordinate R (cm)	Coordinate H (cm)	Value of factor K
A	5	100	4777.648
B	5	96.260	4573.589
C	5.411	82.168	4565.240
D	15	25.644	250475.856
E	15	100	253345.901

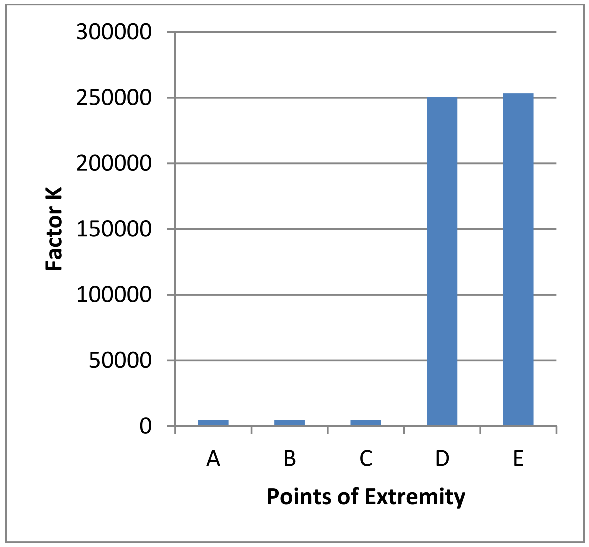

Figure-2. Comparison of factor K at different points of extremity.

The objective function of the design problem is to minimize the Factor K. Referring Table-3 and Figure-2 it can be realized that the Factor K is minimum 4565.240 units at point C and so the point C represents the Optimum Design.

The interpretation of the above can be made as follows:

At point C radius R is 5.411cm and height H is 82.168 cm. Therefore for having minimum value of Factor K i.e. 4565.240 units, the base radius of the conical vessel R should be 5.411 cm while height of the cone should be maintained at 82.168 cm.

This is the ordinary optimum design.

d) Sensitivity robust design

It is cognized that variations in design variables cause variation of the design function. Following the same the variations (tolerances) in radius and height will cause variation in volume, surface area as well as the Factor K (i.e. will cause variation in design function). In the problem statement it is expected that once decided the variation in the value of Factor K should be as minimum as possible because some further critical calculations do involve value of Factor K. It means it is required to have least sensitivity of the Factor K towards variability of the design variables R and H. Thus it is a problem of minimizing the sensitivity of design to the variations of design variables so it is a problem of sensitivity robust optimum design.

The variation in the value of Factor K due to variation or tolerance on R and H is referred as the 'Effect of Transmitted Variation' or as the 'Induced Variation'. For a sensitivity robust design it is required that the design function should be least sensitive to the variability of design variables which is ensured by minimizing the 'Induced Variation'. A design having minimum 'Induced

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Variation' will be least sensitive to the variability and hence it will be termed as a 'Sensitivity Robust Design'.

For the problem in hand, the expected tolerance, both on the base radius R and the height of the vessel H is ± 0.1 cm. As the problem is of minimizing the Factor K, it can be realized that maximum positive tolerance will cause maximum increase in Factor K (i.e. maximum 'Induced Variation') and therefore it will be the worst situation. For sensitivity robust design it is required to minimize the 'Induced Variation' to minimum possible level. Considering the worst case tolerance + 0.1 cm both on the radius R and in the height H if worst case value of Factor K is calculated at different points of extremity then subtracting the original value of Factor K from this worst case value of Factor K the 'Effect of Transmitted variation' or the 'Induced Variation' can be calculated at different points of extremity as tabulated in Table-5. Also after calculating value of 'Induced Variation', dividing it by original value of Factor K, the percentage rise in Factor K or in other words the percentage of 'Induced Variation' can be calculated as shown in Table-6.

Table-4. Value of factor k considering worst case

Point of extremity	Worst case value of R	Worst case value of H	Worst case value of factor K
A	5.1	100.1	4936.892
B	5.1	96.36	4732.677
C	5.511	82.268	4773.663
D	15.1	25.744	258225.613
E	15.1	100.1	261099.197

Table-5. Induced variation in at different points of extremity.

Point of extremity	Worst case value of Factor K	Original value of factor K	Value of induced variation
A	4936.892	4777.648	159.244
B	4732.677	4573.589	159.088
C	4773.663	4565.240	208.423
D	258225.613	250475.856	7749.757
E	261099.197	253345.901	7753.296

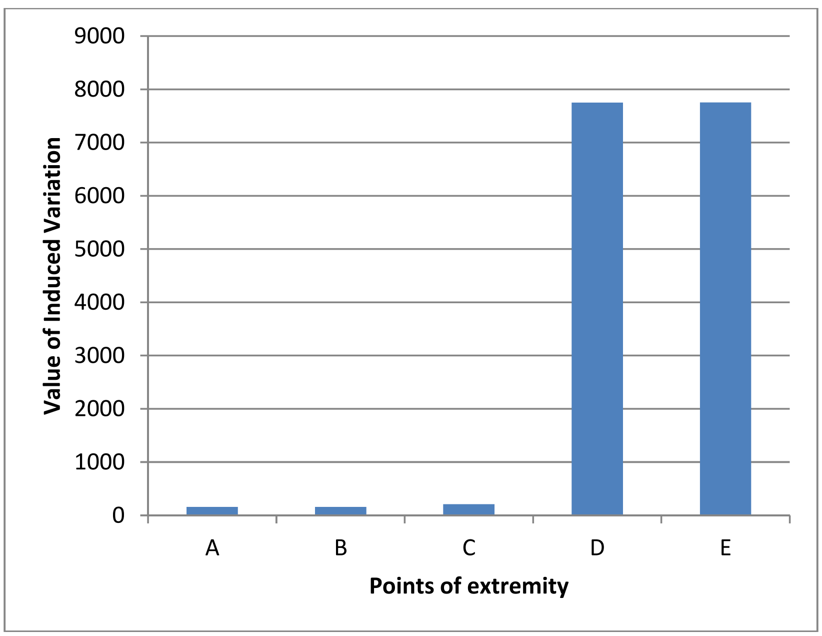

Figure-3. Comparison of value of induced variation in factor K at different points of extremity.

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Table-6. Percent Rise in Factor K or Percentage of Induced variation at different points of extremity.

Point of extremity	Original value of factor K	Value of induced variation	Percent rise in factor K or percentage of induced variation
A	4777.648	159.244	3.333 %
B	4573.589	159.088	3.478 %
C	4565.240	208.423	4.565 %
D	250475.856	7749.757	3.094 %
E	253345.901	7753.296	3.060 %

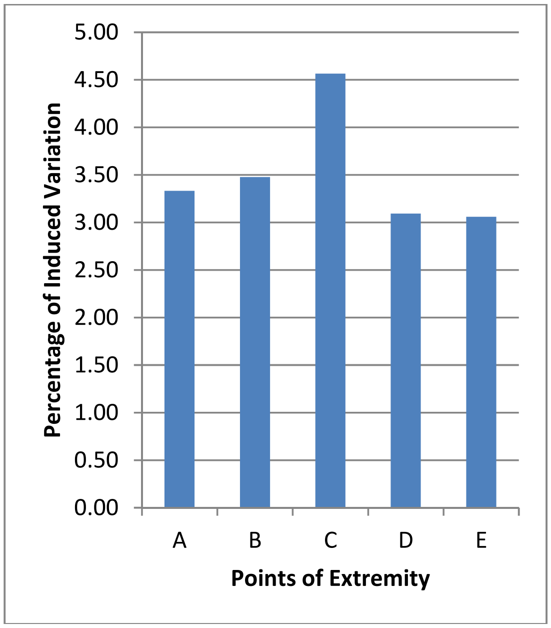

Figure-4. Comparison of percentage of induced variation in factor K at different points of extremity.

Considering the problem statement, it can be realized that while designing (that means while deciding values for radius R and height H) care should be taken so that the values of Factor K is kept at minimum possible level. If this problem is viewed from optimization perspective then to minimize the Factor K is objective function of this optimization problem. Referring Table-3. It can be realized that the point C renders least value of Factor K and hence its co-ordinates signify the (ordinary) optimum design solution for the problem. Taking into account the fact that variations in variable cause variation of design function (i.e. variation/tolerance on radius R and in height H will cause variation of volume, surface area as well as the Factor K) and it is suggested in the problem statement that it is equally rather bit more important to minimize the Induced Variation in value of Factor K. So it is expected to have such a design which is least sensitive towards variability of the design variables (which will be termed as a Sensitivity Robust Design). Referring Table 6. One can note that point E bears least value of percentage Induced Variation and therefore it is least sensitive towards the variation of design variables. Thus point E signifies a 'Sensitivity Robust Design'.

e) Sensitivity robust optimum design

Though point E represents a Sensitivity Robust Design it is not a Sensitivity Robust Optimum Design as it fails to fulfill criterion of minimum value of Factor K and with respect to that aspect point E bears maximum value of Factor K as compared to other points of extremity and hence it is worst design in context of objective function of minimizing the Factor K.

It can be realized that to minimize Factor K and to minimize percentage Induced Variation (Sensitivity of the design) are equally important and any one point of extremity doesn't fulfill both criterion. The point C bears least value of Factor K but maximum value of percentage Induced Variation and therefore it signifies only optimum design. The point E is having least sensitivity (percentage Induced Variation) but it renders maximum value of Factor K and therefore it is only Sensitivity Robust Design and not a Sensitivity Robust Optimum Design. Therefore taking into account both the criterion a tread off is required to be made.

Referring Table-6 it can be noted that at point B the Factor K is marginally greater than that at point C. At point B induced variation is least and percentage Induced Variation though not least (as that at point E) but its value is quite acceptable as against the percentage Induced Variation at point C (at point C it is maximum). Therefore at point B, both the criterions are fulfilled up to a satisfactory level and hence it can be considered that the point B signifies a Sensitivity Robust Optimum Design.

C. Discussion of the results from table-3, table-5 and table-6.

Considering the primary objective function of minimization of the Factor K, the point of extremity, the point C is bearing least value of Factor K and hence it signifies Optimum Design (Ordinary Optimum Design).
Considering the later part of the problem statement and considering minimization of the variation in the calculated value of Factor K as sole objective, point E signifies Sensitivity Robust Design.

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Considering both the objectives and considering the fact that both the objectives are equally important, point B which satisfies both the objectives up to a considerable extent will be declared as the Sensitivity Robust Optimum Design.

Using Table-3 it can be deduced that point C is the optimum but it is the most sensitive and referring Table-6. It can be noted that point E though least sensitive (most robust) it is the worst as far as optimality is concern. Assuming a general tradeoff, point B is finalized as the Robust Optimum Design point (though it is not the true optimum and also it is not the most robust point). It is modest optimum and modest robust point.

D. Loss of optimality

Point C represents Ordinary Optimum Design. Taking into account the issue of variability of design function (i.e. sensitivity of the design towards variability of the design variables) a design having least sensitivity is expected in this case. Quantifying the sensitivity in terms of Induced Variation as the point E bears least value of Induced Variation so it signifies Sensitivity Robust Design.

While making the design a robust one (i.e. moving away from point C towards point E), the Factor K increases from 4565.240 to 253345.901. Primary objective function of the design is to minimize the Factor K and therefore the effective increase in value of Factor K (while making the design robust) indicates the loss of optimality. Thus while achieving robustness in the design, optimality is lost up to a certain extent and this loss of optimality can be considered as the cost paid for achieving robustness. Generally the optimality is sacrificed up to a certain extent for the sake of less sensitivity or sensitivity robustness of the design.

Practically a tradeoff is made between the robustness achieved in the design and the loss of optimality. This tradeoff is made on the basis of degree of robustness required and the loss of optimality affordable i.e. on the basis of cost of loss of optimality and need of robustness in the design. In this case a prudential tradeoff is made by selecting point B as the design point and declaring it as a point signifying Sensitivity Robust Optimum Design. Choice of point B is appropriate as at this point the loss of optimality is marginal whereas the gain in Sensitivity Robustness is substantial.

3. CONCLUSIONS

A Robust Optimum Design is that Optimum Design which tolerates the variations where variations are expected deviation of design variables and parameters from their set values.

Variations in design variables and parameters get transmitted to design function. Design function means the objective function of the design, the functional requirements of the design and the undesirable effects in the design. The variation getting transmitted to the design function is called as Induced Variation. Induced variation may cause violation of constraint boundaries and may cause improper functioning or even failure due to that. Thus Induced variation is of concern.

An Optimum Design is generally rigid and it does not tolerate Induced Variation (which is due to variability of the design variables and parameters). If the effect of Induced Variation is minimized/nullified in such a way that despite variations the design is feasible and no improper functioning or no failure is ensured then the design can be considered to be capable of tolerating the variations and it is a 'Feasibility Robust Optimum Design' [1], [2] & [3]. If the effect of Induced Variation is reduced to such an extent that variation of design function is hardly noticeable and hardly matters then as the sensitivity of the design is minimized, it is assumed that the design tolerates variations and hence it is robust. Such a design is called as a 'Sensitivity Robust Optimum Design' [4] & [5].

In this illustration a fictitious example of designing a conical vessel is discussed and it is illustrated that how graphical method of optimization can be employed for achieving ordinary Optimum Design, Sensitivity Robust Design and Sensitivity Robust Optimum Design.

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