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GRAVITATIONAL SEARCH ALGORITHM: R IS BETTER THAN R^2?

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ABSTRACT

Gravitational Search Algorithm (GSA) is a metaheuristic population-based optimization algorithm inspired by the Newtonian law of gravity and law of motion. Ever since it was introduced in 2009, GSA has been employed to solve various optimization problems. Despite its superior performance, GSA has a fundamental problem. It has been revealed that the force calculation in GSA is not genuinely based on the Newtonian law of gravity. Based on the Newtonian law of gravity, force between two masses in the universe is inversely proportional to the square of the distance between them. However, in the original GSA, R is used instead of R^2. In this paper, the performance of GSA is re-evaluated considering the square of the distance between masses, R^2. The CEC2014 benchmark functions for real-parameter single objective optimization problems are employed in the evaluation. An important finding is that by considering the square of the distance between masses, R^2, significant improvement over the original GSA is observed provided a large gravitational constant should be used at the beginning of the optimization process.

Keywords: gravitational search algorithm, newtonian law of gravity, law of motion.

INTRODUCTION

Gravitational Search Algorithm (GSA) has been firstly introduced by Rashedi *et al.* in 2009 [1]. It is a metaheuristic population-based optimization algorithm which is inspired by the Newtonian law of gravity and law of motion. In GSA, fitness is translated into mass and interaction between agents is simulated based on the Newtonian Law of Gravity and Law of Motion.

However after three years GSA was introduced, Gauci *et al.* [2] has found an inconsistency used of gravitational formulation in GSA. They have proved theoretically that GSA was indeed not genuinely based on Newtonian law of gravity. Specifically, in the calculation of force, distance R is employed instead of R^2. However, the main reason of this has been explained in the first paper of GSA. The original author stated in [1], original gravitational formulation was not used because of poor experimental result.

Therefore, in this paper, we re-evaluate the performance of standard GSA with distance R, using CEC2014 benchmark dataset. Then, we propose the use of square of distance R^2, in the calculation of force which we denoted as GSAR2. We also investigate the performance of GSAR2 algorithm by varying the value of initial gravitational constant, G_0. The performance is then analyzed statistically.

GRAVITATIONAL SEARCH ALGORITHM

In GSA, agents are considered as an object and their performance are expressed by their masses. The position of particle is corresponding to the solution of the problem. Consider a population consisted N quantity of agents, so the position of ith agent can be presented by:

$$X_i = \left(x_i^1 \dots x_i^d \dots x_i^n\right) \; for \; i = 1,2,\dots,N \tag{1}$$

The mass of ith particle at time t is derived from Eqn. (2) and Eqn. (3), denoted as $M_i(t)$.

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{2}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{3}$$

where N is a population size, $m_i(t)$ is an intermediate variable in agent mass calculation, $fit_i(t)$ is the fitness value of ith agent at time t, $best(t)$ and $worst(t)$ denote the best and the worst fitness value of the population at time t. The best and the worst fitness for the case of minimization problem are defined as follows;

$$\begin{aligned} best(t) &= \min_{j\in\{1,\dots,N\}} fit_j(t) \\ worst(t) &= \max_{j\in\{1,\dots,N\}} fit_j(t) \end{aligned} \tag{4}$$

whereas for maximization problem,

$$\begin{aligned} best(t) &= \max_{j\in\{1,\dots,N\}} fit_j(t) \\ worst(t) &= \min_{j\in\{1,\dots,N\}} fit_j(t) \end{aligned} \tag{5}$$

At specific time "t", the force acting on agent "i" from agent "j" in dth dimension can be represented as the following:

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t)\times M_{ij}(t)}{R_{ij}(t)+\varepsilon}\left(x_j^d(t) - x_i^d(t)\right) \quad (6)$$

where $M_{pi}(t)$ is the passive gravitational mass of agent "i", $M_{aj}(t)$ is the active gravitational mass of agent "i", $G(t)$ is the gravitational constant, ε is a small constant, and $R_{ij}(t)$ is the Euclidian distance between agent "i" and "j". The distance is calculated using a standard formula as follow;

$$R_{ij}(t) = \left\|X_i(t), X_j(t)\right\|_2 \quad (7)$$

while gravitational constant, $G(t)$, is defined as a decreasing function of time, which is set to G_0 at the beginning and decreases exponentially towards zero with lapse of time [3].

$$G(t) = G_0 \times e^{-\alpha\frac{t}{t_{max}}} \quad (8)$$

To give a stochastic characteristic to GSA, the total force acted on agent "i" in "d" dimension is a randomly weighted sum of dth components of the forces exerted from other agents;

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_j F_{ij}^d(t) \quad (9)$$

where $rand_i$ is a random number in the interval of [0,1].

According to law of motion, the current velocity of any mass is equal to the sum of the fraction of its previous velocity and the variation in the velocity. Variation or acceleration of any mass is equal to the force acted on the system divided by mass of inertia [3], which is shown in the following formula.

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \; for\; M_{ai} = M_{pi} = M_{ii} \quad (10)$$

Therefore, the new agent velocity and position are calculated using these equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \quad (11)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (12)$$

Finally, the next iteration is executed until the maximum number of iterations, t_{max}, is reached. In summary, the principle of standard GSA is shown in Figure-1.

GSA IS NOT GENUINELY FOLLOWS THE NEWTONIAN GRAVITATIONAL LAW

Newton stated that "Every particle in the universe attract every other particle with a force that is directly proportional to the square of the distance between them" [4]. This definition is known as gravitational force and it is formulated as:

$$F = G\frac{M_1 M_2}{R^2} \quad (13)$$

In GSA, the calculation of force is also based on this equation. However, as shown in Eqn. (6), distance R, is used as the denominator instead of R^2. Let $\varepsilon = 0$, then

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t)\times M_{ij}(t)}{R_{ij}(t)}\left(x_j^d(t) - x_i^d(t)\right) \quad (14)$$

since $R_{ij}(t) = x_j^d(t) - x_i^d(t)$, therefore,

$$F_{ij}^d(t) = G(t) \times M_{pi}(t) \times M_{ij}(t) \quad (15)$$

which clearly shows that the force $F_{ij}(t)$ is not influenced by the distance between agent i and j. Thus, the original GSA is not genuinely follows the Newtonian gravitational law.

In this paper, we follow genuinely the Newtonian gravitational law and use the square of distance, R^2, in the calculation of force. The performance of GSAR2 with different value of initial gravitational constant is investigated as well.

EXPERIMENTS

The parameter setting for all experiments is tabulated in Table-1. Different value of G_0, $G_0 = 10^1$ until $G_0 = 10^{15}$ were tested in experiments for GSAR2.

In this study, 30 standard benchmark functions from CEC2014 test functions [5] shown in Table-2 were used throughout the experiment. These benchmark functions consist of the shifted, rotated, expanded and combined classical test function. They are categorized into three four groups; unimodal, multimodal, hybrid, and composite function.

RESULT AND DISCUSSION

Convergence curves of the two variations of GSA, which is the original GSA and GSA that employs square of distance between masses (GSAR2), are shown in Figure-2 to Figure-5. For GSAR2, $G_0=10^9$ is used. These results show that generally better performance can be obtained even though square of distance between masses is used.

Analysis of convergence curves of GSAR2 with different G_0 are shown in Figure-6 to Figure-10. These figures show that solutions can be improved faster and convergence rate is better if smaller G_0 is used. However, there is no guarantee that small G_0 produces better result.

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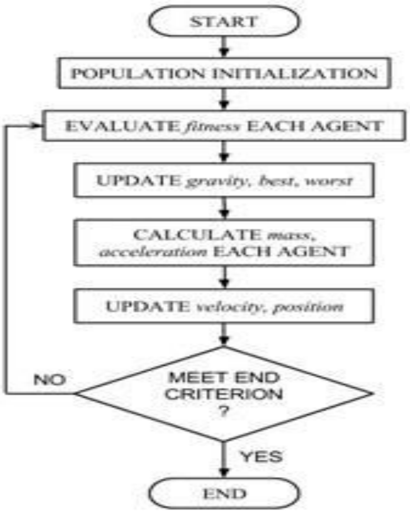

Figure-1. General principle of standard GSA.

Table-1. Parameter setting used in all experiments.

Parameter	Value
Number of agents, N	100
Number of iterations, t	2000
Number of dimensions, D	50
Number of runs, t_{max}	50
Search range	[100,-100]
Alpha, α	20

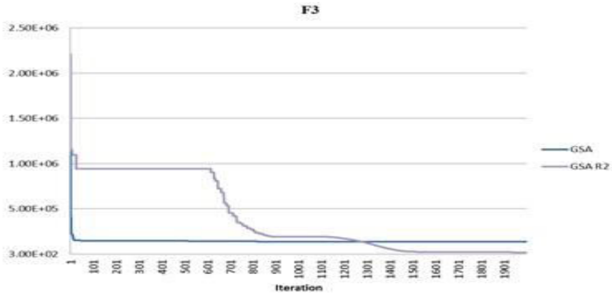

Figure-2. Convergence curve for function 3, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.

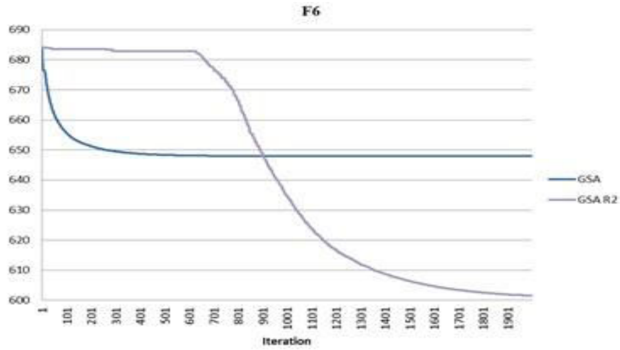

Figure-3. Convergence curve for function 6, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.

Table-2. CEC 2014 benchmark functions.

Function Type	Function ID	Function Description	Ideal Fitness
Unimodal Function	F1	Rotated High Conditioned Elliptic Function	100
	F2	Rotated Bent Cigar Function	200
	F3	Rotated Discus Function	300
Simple Multimodal Function	F4	Shifted and Rotated Rosenbrock's Function	400
	F5	Shifted and Rotated Ackley's Function	500
	F6	Shifted and Rotated Weierstrass Function	600
	F7	Shifted and Rotated Griewank's Function	700
	F8	Shifted Rastrigin's Function	800
	F9	Shifted and Rotated Rastrigin's Function	900
	F10	Shifted Schwefel's Function	1000
	F11	Shifted and Rotated Schwefel's Function	1100
	F12	Shifted and Rotated Katsuura Function	1200
	F13	Shifted and Rotated HappyCat Function	1300
	F14	Shifted and Rotated HGBat Function	1400
	F15	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	1500
	F16	Shifted and Rotated Expanded Scaffer's F6 Function	1600
Hybrid Function	F17	Hybrid Function 1 (N=3)	1700
	F18	Hybrid Function 2 (N=3)	1800
	F19	Hybrid Function 3 (N=4)	1900
	F20	Hybrid Function 4 (N=4)	2000
	F21	Hybrid Function 5 (N=5)	2100
	F22	Hybrid Function 5 (N=5)	2200
Composite Function	F23	Composition Function 1 (N=5)	2300
	F24	Composition Function 2 (N=3)	2400
	F25	Composition Function 3 (N=3)	2500
	F26	Composition Function 4 (N=5)	2600
	F27	Composition Function 5 (N=5)	2700
	F28	Composition Function 6 (N=5)	2800
	F29	Composition Function 7 (N=3)	2900
	F30	Composition Function 8 (N=3)	3000

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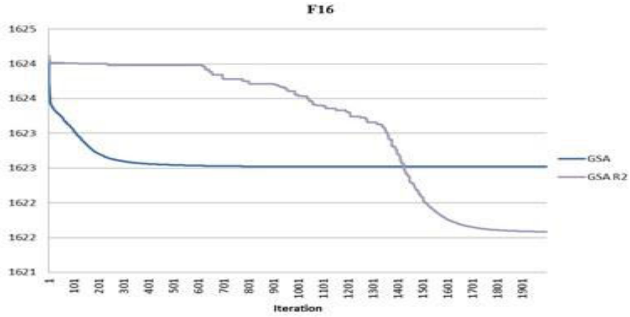

Figure-4. Convergence curve for function 16, in which, $G_0=10^2$ is used for original GSA and $G_0=10^9$ is used for GSAR2.

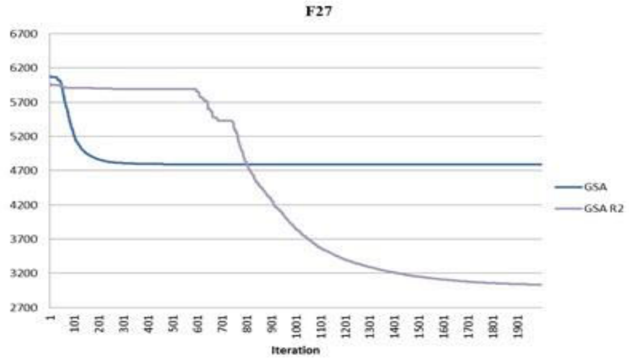

Figure-5. Convergence curve for function 27, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.

According to inferential statistic, hypothesis testing can be used to obtain inferences about one or more algorithms from given sample. This can be achieved by defining two types of hypothesis, the null hypothesis H_0 and the alternative hypothesis H_1. The null hypothesis is a statement of no effect or no difference, whereas the alternative hypothesis represents significant difference between algorithms.

Friedman's test is an omnibus test which can be used to carry out these types of comparison. It allows us to detect differences considering the global set of algorithms. Once Friedman's test rejects the null hypothesis, we can proceed with a post-hoc test in order to find the concrete pairwise comparisons which produce differences.

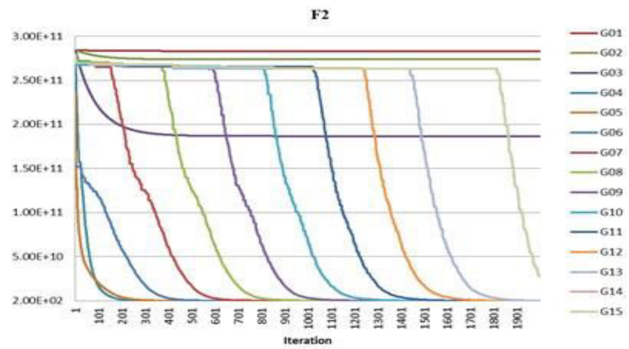

Figure-6. Convergence curves of different G0 values for function 27.

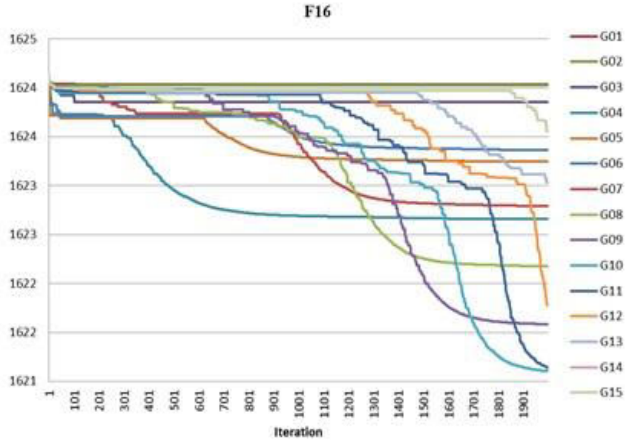

Figure-7. Convergence curves of different G0 values for function 16.

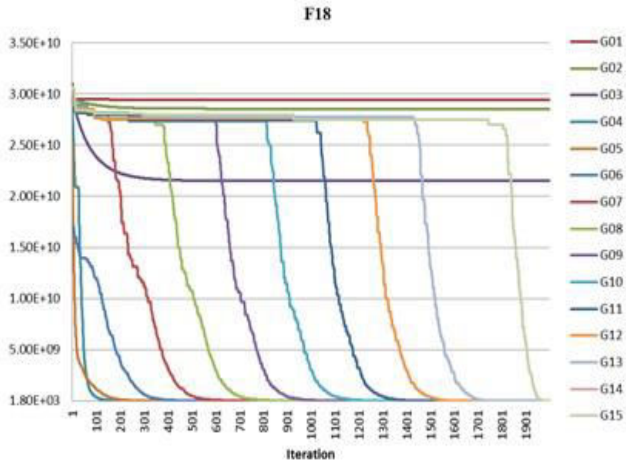

Figure-8. Convergence curves of different G0 values for function 18.

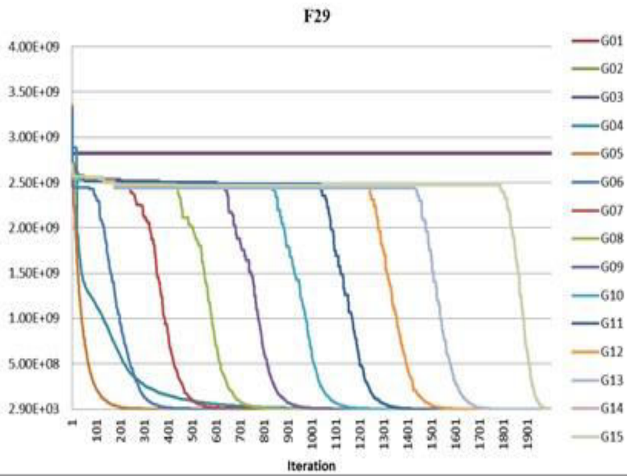

Figure-9. Convergence curves of different G0 values for function 29.

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Table-3. Friedman test result for variant of G0 value.

Function	GSA with square of the distance between masses (GSA R2)														
	$G_0=10^1$	$G_0=10^2$	$G_0=10^3$	$G_0=10^4$	$G_0=10^5$	$G_0=10^6$	$G_0=10^7$	$G_0=10^8$	$G_0=10^9$	$G_0=10^{10}$	$G_0=10^{11}$	$G_0=10^{12}$	$G_0=10^{13}$	$G_0=10^{14}$	$G_0=10^{15}$
1	8732661254	8506448153	6865231739	2511952.1	1506554	1281282	1132635	1137142	1289971	1781130	2751339	5522629	14421108	52511993.8	537606682.7
2	2.83273E+11	2.73774E+11	1.8679E+11	7104.7445	3440.425	3707.562	3729.365	6416.709	22565.14	182438	1918904	18770454	184440486	1815735743	27739677412
3	1689294.335	1733449.044	1525028.31	114058.96	87550.64	52177.54	22883.16	18141.65	17514.3	17891.38	20236.09	27013.8	28286.319	37330.5695	157738.8777
4	119488.5411	113268.978	63365.6885	669.82102	547.3768	534.1597	520.6037	512.5391	515.0092	529.9766	541.9242	568.8776	623.39436	841.059843	3864.257868
5	521.43634	521.4257227	521.429725	519.99975	519.9999	519.9999	520.0001	520.0019	520.0216	520.2316	521.0816	521.4196	521.4338	521.422384	521.435732
6	683.8335855	683.8006746	680.889041	631.69971	603.8811	602.6572	601.9596	601.5161	601.6171	602.3055	604.1438	607.6323	614.83	631.166609	662.6122999
7	3344.47661	3255.074092	2447.35121	700	700	700	700.0004	700.0021	700.0225	700.2629	700.9354	701.1698	702.59033	715.78177	948.8673236
8	1725.12674	1708.4137	1564.91326	1057.6139	861.5879	849.1311	844.5941	839.1623	836.8605	837.9742	838.6608	853.3049	921.39109	1104.22403	1352.220486
9	2073.113791	2048.117556	1804.91726	1160.8371	943.3205	936.0374	934.565	931.0835	931.133	934.9526	938.3279	959.2188	1076.904	1341.62882	1516.43828
10	17380.29708	17325.03038	16770.4643	7588.2786	5369.088	4959.903	4492.397	4141.226	3900.216	3803.741	3513.073	3424.28	4325.9472	8920.44787	14887.70658
11	17770.8845	17774.70851	17460.3442	8290.2016	4289.283	3786.806	3642.516	3473.845	3496.616	3369.126	3480.942	4013.827	6885.6767	13869.721	17030.29615
12	1206.628195	1206.894843	1207.03495	1200.0011	1200.001	1200.001	1200.002	1200.003	1200.007	1200.023	1200.114	1200.882	1203.5324	1206.53505	1207.007219
13	1312.296504	1312.014347	1309.36737	1300.3592	1300.253	1300.256	1300.248	1300.255	1300.28	1300.359	1300.484	1300.666	1300.8531	1301.12134	1303.567935
14	2114.434403	2089.320553	1861.75842	1400.3077	1400.478	1400.497	1400.5	1400.5	1400.492	1400.467	1400.466	1400.728	1401.0573	1402.71712	1471.34932
15	135512671.5	117340634.1	20547810.2	1510.5936	1505.196	1506.854	1506.658	1506.498	1506.237	1506.523	1513.366	1539.534	1546.4017	1554.35725	24790.23983
16	1624.032145	1624.068633	1623.90397	1622.6613	1623.247	1623.366	1622.791	1622.174	1621.583	1621.104	1621.139	1621.774	1623.0767	1623.27904	1623.686076
17	1114799377	1090382363	956772098	239017.96	135540.3	112017.1	109852.9	108352.8	279582.1	295618.9	324646.7	685530.9	1023041.8	2761829.76	22716442.25
18	29441038983	28556747406	2.1504E+10	3489.5086	2233.825	2199.409	2211.236	2273.758	2525.324	2470.741	2519.964	2338.366	2633.9387	2742.06376	7931603.248
19	6802.013661	6608.619455	5255.46068	1929.4708	1961.237	1969.123	1965.455	1964.703	1962.814	1961.57	1966.128	1971.106	1971.5503	1972.67694	2011.130942
20	19843113.88	16878522.37	4941606.83	32527.539	30702.06	29231.62	29664.45	26704.9	22549.41	21087.68	23631.41	25516.52	36368.637	47399.2724	142994.0184
21	418358577.7	404233380.4	268429891	374185.25	202344.1	180931.8	162851.4	146788.8	140888.7	185657.3	251913	289969.3	334712.08	614614.432	3190329.174
22	766218.7422	723181.9558	551322.556	3833.7855	3757.863	3552.14	3365.425	3164.844	2923.023	2714.538	2766.163	2686.664	2799.0476	2945.26427	4923.765685
23	6294.418892	6109.699109	4670.55103	2516.482	2656.43	2652.985	2650.515	2649.065	2648.032	2647.637	2648.181	2649.523	2650.3922	2656.21733	2703.859884
24	3311.147434	3286.132418	3032.13398	2600.1315	2658.279	2657.539	2656.993	2656.726	2657.475	2656.837	2658.303	2661.062	2671.8596	2703.48088	2794.352455
25	3276.140837	3250.64416	2982.63517	2700.0002	2700.001	2704.24	2712.376	2720.117	2719.03	2715.018	2713.744	2714.097	2716.9106	2723.18453	2760.631227
26	3329.691419	3305.818183	3014.08744	2800.0534	2800.093	2800.133	2800.109	2800.123	2798.024	2798.048	2798.25	2785.554	2791.5313	2785.81649	2782.236792
27	6155.768633	6179.73393	7044.51333	4341.1057	3028.074	3009.722	3010.903	3019.961	3032.667	3055.574	3105.057	3196.33	3387.0514	3802.90628	4563.212503
28	19689.97114	19947.74393	24033.8342	6618.5917	3421.272	3196.162	3188.55	3169.835	3169.596	3193.983	3303.21	3574.232	4661.7652	5790.88651	10065.97726
29	2834996594	2939130498	3639061933	24012.206	12057.08	12070.83	10780.81	8168.29	6029.955	5527.441	5116.527	5321.048	6219.6934	9430.76455	3232641.315
30	59488559.17	60313586.71	803640658	148393.95	117847.5	100906.5	79926.06	61653.13	53467.3	50826.33	53365.54	59118.85	62242.152	102745.936	417009.8984
Avg Rank	**14.53**	**13.97**	**13.30**	**7.00**	**5.83**	**5.67**	**4.73**	**4.13**	**4.00**	**4.13**	**5.53**	**6.73**	**8.63**	**10.03**	**11.77**

Table-4. G0 variant for post-hoc comparison using Holm procedure.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
105	G01 vs. G09	9.122134	0	0.000476
104	G01 vs. G08	9.006664	0	0.000481
103	G01 vs. G10	9.006664	0	0.000485
102	G02 vs. G09	8.631387	0	0.00049
101	G02 vs. G08	8.515916	0	0.000495
100	G02 vs. G10	8.515916	0	0.0005
99	G01 vs. G07	8.487049	0	0.000505
98	G03 vs. G09	8.054036	0	0.00051
97	G02 vs. G07	7.996301	0	0.000515
96	G03 vs. G08	7.938566	0	0.000521
95	G03 vs. G10	7.938566	0	0.000526
94	G01 vs. G11	7.794229	0	0.000532
93	G01 vs. G06	7.72206	0	0.000538
92	G01 vs. G05	7.534421	0	0.000543
91	G03 vs. G07	7.418951	0	0.000549
90	G02 vs. G11	7.303481	0	0.000556
89	G02 vs. G06	7.231312	0	0.000562
88	G02 vs. G05	7.043673	0	0.000568
87	G01 vs. G12	6.754998	0	0.000575
86	G09 vs. G15	6.726131	0	0.000581
85	G03 vs. G11	6.726131	0	0.000588
84	G03 vs. G06	6.653962	0	0.000595
83	G08 vs. G15	6.610661	0	0.000602
82	G10 vs. G15	6.610661	0	0.00061
81	G01 vs. G04	6.480757	0	0.000617
80	G03 vs. G05	6.466323	0	0.000625
79	G02 vs. G12	6.26425	0	0.000633
78	G07 vs. G15	6.091045	0	0.000641
77	G02 vs. G04	5.990009	0	0.000649
76	G03 vs. G12	5.6869	0	0.000658
75	G03 vs. G04	5.412659	0	0.000667
74	G11 vs. G15	5.398225	0	0.000676
73	G06 vs. G15	5.326056	0	0.000685
72	G09 vs. G14	5.22502	0	0.000694
71	G05 vs. G15	5.138417	0	0.000704
70	G01 vs. G13	5.10955	0	0.000714
69	G08 vs. G14	5.10955	0	0.000725
68	G10 vs. G14	5.10955	0	0.000735
67	G02 vs. G13	4.618802	0.000004	0.000746
66	G07 vs. G14	4.589935	0.000004	0.000758
65	G12 vs. G15	4.358995	0.000013	0.000769
64	G04 vs. G15	4.084753	0.000044	0.000781
63	G03 vs. G13	4.041452	0.000053	0.000794
62	G09 vs. G13	4.012584	0.00006	0.000806
61	G01 vs. G14	3.897114	0.000097	0.00082
60	G11 vs. G14	3.897114	0.000097	0.000833
59	G08 vs. G13	3.897114	0.000097	0.000847
58	G10 vs. G13	3.897114	0.000097	0.000862
57	G06 vs. G14	3.824946	0.000131	0.000877
56	G05 vs. G14	3.637307	0.000276	0.000893
55	G02 vs. G14	3.406367	0.000658	0.000909
54	G07 vs. G13	3.377499	0.000731	0.000926
53	G12 vs. G14	2.857884	0.004265	0.000943
52	G03 vs. G14	2.829016	0.004669	0.000962

Table-3 shows the overall experimental results for Friedman procedure obtained in this study. For the case of GSAR2, value of G0=109 provides the best average ranking among others. These results were subjected to post-hoc test using Holm procedures and the results are shown in Table-4. According to Holm's procedure, hypothesis that have an adjusted p-value less or equal to 0.001887 are rejected.

Table-5. GSA Original vs GSAR2 with G0=109 Wilcoxon test result.

Function	GSA ORI	GSA R2	SIGN	ABS	R	SIGN R
1	14775830.9	1289971.341	1	13485859.58	28	28
2	22443764.2	22565.13575	1	22421199.09	29	29
3	138080.202	17514.3008	1	120565.9014	25	25
4	878.734707	515.0091718	1	363.7255	16	16
5	519.999717	520.0215943	-1	0.0219	2	-2
6	647.955355	601.6171107	1	46.3382	10	10
7	702.097125	700.0225056	1	2.0746	7	7
8	1076.49811	836.8604789	1	239.6376	13	13
9	1250.69963	931.132958	1	319.5667	15	15
10	8193.16657	3900.216001	1	4292.9506	21	21
11	9275.68745	3496.615988	1	5779.0715	22	22
12	1200.00289	1200.007415	-1	0.0045	1	-1
13	1300.47788	1300.280122	1	0.1978	4	4
14	1400.29839	1400.49231	-1	0.1939	3	-3
15	1765.90409	1506.23739	1	259.6667	14	14
16	1622.52317	1621.582546	1	0.9406	5	5
17	2181643.85	279582.1133	1	1902061.733	27	27
18	69338904.1	2525.323747	1	69336378.74	30	30
19	1944.0205	1962.814272	-1	18.7938	8	-8
20	59215.9615	22549.41379	1	36666.5477	23	23
21	1844950.35	140888.6913	1	1704061.661	26	26
22	4133.86178	2923.022917	1	1210.8389	17	17
23	2500	2648.032109	-1	148.0321	12	-12
24	2600.09343	2657.475362	-1	57.3819	11	-11
25	2700	2719.030494	-1	19.0305	9	-9
26	2800.08141	2798.023698	1	2.0577	6	6
27	4789.01228	3032.667359	1	1756.3449	18	18
28	6083.88723	3169.595938	1	2914.2913	19	19
29	3100.15831	6029.954977	-1	2929.7967	20	-20
30	3200.01244	53467.29837	-1	50267.2859	24	-24

In other words, G0=101, G0=102, G0=103, G0=1013, G0=1014, and G0=1015 are significantly different compared to G0=109 which was highlighted in Table-4. The rest of G0 value has no significant difference between each other. However, based on the average ranking, result of G0=109 is chosen for the comparison with the original GSA in pairwise Wilcoxon test. According to the result of the Wilcoxon test shown in Table-5, by using p-value equal to 0.05, the Z-value obtained is -2.931. Based on normal distribution curve it shows p-value for -2.932 is equal to 0.00338 which is smaller than 0.05. So it can be concluding the GSAR2 not only better than the original GSA in terms of performance, but also significant difference exists between these two algorithms.

CONCLUSIONS

The original GSA algorithm is not genuinely follows the Newtonian gravitational law. In this paper, by correcting the force of calculation in original GSA and investigating various initial gravitational constants G0, GSAR2 has been proposed. It is found that the GSAR2 not only superior to the original GSA, but most importantly, GSAR2 follows more closely to the Newtonian gravitational law.

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